

2020 年微分方程小考

(線習用，每小題 15 分)

1. Find the solution of the following DEs:

(a) $y''(x) + 3y'(x) + 2y(x) = \frac{1}{1+e^x}$

(b) $4(x+1)^2 y''(x) + y(x) = \ln(x+1)^2$

2. Suppose that the weight of an object is 10 kgw. Initially, the location of the object is 0 and the velocity is 1m/s. Express the location of the object in terms of time t when the force is $20\exp(-t)$ newton and the friction is $2a$ newton if the velocity is a m/s.

Use higher-order DEs to solve these problems.

3. (a) Find the power series solution of

$$y''(x) - 2xy'(x) + 8y(x) = 0, \quad y(0) = 3, \quad y'(0) = 0$$

(b) Use the Frobenius method to solve

$$9x^2 [y''(x) + y'(x)] + 2y(x) = 0$$

4. Suppose that

$$P_0(x) = 1, \quad P_1(x) = x + c$$

Find c such that $P_1(x)$ is orthogonal to $P_0(x)$ with respect to the weight function

$$w(x) = \exp(-x)$$

and the given interval of $x \in [0, 2]$.

5. Find the Fourier series of

$$f(x) = \begin{cases} 1+x, & -1 < x < 0 \\ 1-x, & 0 < x < 1 \end{cases}$$