**Summary Tables for Formulas**

**(1) The Methods to Solve First Order Differential Equations**

|  |  |  |
| --- | --- | --- |
| Name of Methods | Suitable Conditions | Key Formula of the Method |
| (1) Direct Integral |  |  |
| (2) Separable Variables |  |  |
| (3) Linear DE  |  |  |
| (4) Exact Equation |  | First, solve , then solve  (or exchange the order) |
| (5) Exact Equation (Integration Factor I) |  is independent of *x* | , solving |
| (6) Exact Equation (Integration Factor II) |  is independent of *y* | , solving  |
| (7) Homogeneous equation |  | Set *u* = *y*/*x,* (*y* = *xu*)   |
| (8) Bernoulli’s Equation |  | Set *u* = *y*1–*n*   |
| (9) *Ax* + *By* + *C*  |  | Set *u* = *Ax + By + c*    |

**(2) The Methods to Solve Higher Order Linear Differential Equations**

|  |
| --- |
| (A) Solving the Complementary Function for Linear DEs |
| Name of Methods | Suitable Conditions | Key Formula of the Method |
| (1) Reduction of Order | (i) linear, (ii) 2nd order, (iii) one nontrivial solution *y*1(*x*) has been known | where *P*(*x*) is the coefficient of in the *standard form*.  |
| (2) Auxiliary Function  | (i) linear, (ii) constant coefficients |  |
| (a) If *mq* is a root |  is one of the independent solutions.  |
| (b) If *mq* is a root with multiplicity *k*  |  are *k* of the independent solutions  |
| (c) If *α* ± *jβ* are the roots |  are two of the independent solutions |
| (d) If *α* ± *jβ* are the roots with multiplicity *k*  | are 2*k* of the independent solutions |
| (B) Solving the Particular Solution for Linear DEs  |
| Name of Methods | Suitable Conditions | Key Formula of the Method |
| (1) Form Rule | (i) linear, (ii) constant coefficients, (iii) *g*(*x*), *g*'(*x*), *g*''(*x*), …. have finite number of terms  | See PowerPoint Page 195 |
| (2) Annihilator Method | (i) linear, (ii) constant coefficients, (iii) *g*(*x*), *g*'(*x*), *g*''(*x*), …. have finite number of terms | If , original DE: , , Particular solutions:Satisfy , but  |
| *g*(*x*) | Annihilator |
|  |  |
|  |  |
| *g*1(*x*) + *g*2(*x*) + …… + *gk*(*x*) |  if*Lh*[*gh*(*x*)] = 0 |

**(3) Formulas for Trigonometric and Hyperbolic Functions**

|  |  |
| --- | --- |
| (1) cos(*x*) =  |  |
| (2) sin(*x*) =  |  |
| (3) cosh(*x*) = |  |
| (4) sinh(*x*) = |  |
| (5) cos(*a*+*b*) =  | cos(*a*)cos(*b*) − sin(*a*) sin (*b*) |
| (6) sin(*a*+*b*) = | sin(*a*)cos(*b*) + cos(*a*)sin (*b*) |
| (7) cos(2*a*) =  | cos2(*a*) − sin2(*a*)  |
| (7a) cos2(*a*) =  | (cos(2*a*) + 1)/2  |
| (7b) sin2(*a*) = | (1 − cos(2*a*))/2 |
| (8) sin(2*a*) = | 2 sin(*a*) cos(*a*) |
| (9) sinh(0) = | 0 |
| (10) cosh(0) = | 1 |
| (11)  | ,  |
| (12)  | ,  |

**(4) Integrals**

|  |  |
| --- | --- |
| (1) 1/*x* | ln|*x*| + *c* |
| (2) cos(*x*) | sin(*x*) + *c* |
| (3) sin(*x*) | –cos(*x*) + *c* |
| (4) tan(*x*) | –ln|cos(*x*)| + *c* |
| (5) cot(*x*) | ln|sin(*x*)| + *c* |
| (6) *ax* | *ax*/ln(*a*) + *c* |
| (7)  |  |
| (8)  |  |
| (9)  |  |
| (10) *x* *eax* |  |
| (11) *x*2 *eax* |  |