

Summary Tables for Formulas

(1) The Methods to Solve First Order Differential Equations

Name of Methods	Suitable Conditions	Key Formula of the Method
(1) Direct Integral	$\frac{dy}{dx} = f(x)$	$y(x) = \int f(x) dx + c$
(2) Separable Variables	$\frac{dy}{dx} = g(x)h(y)$	$\frac{dy}{h(y)} = g(x) dx$
(3) Linear DE	$\frac{dy}{dx} + P(x)y = f(x)$	$\frac{d}{dx} \left[e^{\int P(x) dx} y \right] = e^{\int P(x) dx} f(x)$
(4) Exact Equation	$M(x, y) dx + N(x, y) dy$	First, solve $\frac{\partial f}{\partial x} = M(x, y)$, then solve $\frac{\partial f}{\partial y} = N(x, y)$ (or exchange the order)
(5) Exact Equation (Integration Factor I)	$M(x, y) dx + N(x, y) dy$ $\frac{M_y - N_x}{M}$ is independent of x	$\mu(y) = e^{\int \frac{(N_y - M_x)}{M} dy}$, solving $\mu(y) M(x, y) dx + \mu(y) N(x, y) dy$
(6) Exact Equation (Integration Factor II)	$M(x, y) dx + N(x, y) dy$ $\frac{M_y - N_x}{N}$ is independent of y	$\mu(x) = e^{\int \frac{(M_y - N_x)}{N} dx}$, solving $\mu(x) M(x, y) dx + \mu(x) N(x, y) dy$
(7) Homogeneous equation	$M(x, y) dx + N(x, y) dy$ $M(tx, ty) = t^\alpha M(x, y)$ $N(tx, ty) = t^\alpha N(x, y)$	Set $u = y/x$, ($y = xu$) $dy = u dx + x du$
(8) Bernoulli's Equation	$\frac{dy}{dx} = -P(x)y + f(x)y^n$	Set $u = y^{1-n}$ $\frac{du}{dx} = \frac{1}{1-n} u^{1-n} \frac{du}{dx}$
(9) $Ax + By + C$	$\frac{dy}{dx} = f(Ax + By + C)$	Set $u = Ax + By + c$ $B dy = du - A dx$

(2) The Methods to Solve Higher Order Linear Differential Equations

(A) Solving the Complementary Function for Linear DEs		
Name of Methods	Suitable Conditions	Key Formula of the Method
(1) Reduction of Order	(i) linear, (ii) 2 nd order, (iii) one nontrivial solution $y_1(x)$ has been known	$y_2(x) = y_1(x) \int \frac{e^{-\int P(x)dx}}{y_1^2(x)} dx$ <p>where $P(x)$ is the coefficient of y' in the <i>standard form</i>.</p>
(2) Auxiliary Function	(i) linear, (ii) constant coefficients	$\sum_{n=0}^N a_n y^{(n)}(x) = 0 \rightarrow \sum_{n=0}^N a_n m^n = 0$
(B) Solving the Particular Solution for Linear DEs		
$\sum_{n=0}^N P_n(x) y^{(n)}(x) = g(x)$		
Name of Methods	Suitable Conditions	Key Formula of the Method
(1) Form Rule	(i) linear, (ii) constant coefficients, (iii) $g(x)$, $g'(x)$, $g''(x)$, ... have finite number of terms	See PowerPoint Page 198 to find y_p
(2) Annihilator	(i) linear, (ii) constant coefficients, (iii) $g(x)$, $g'(x)$, $g''(x)$, ... have finite number of terms	<p>If $L_1 g(x) = 0$,</p> <p>original DE: $Ly = g$, $L = \sum_{n=0}^N a_n D^n$,</p> <p>Particular solutions: Satisfy $L_1 Ly = 0$, but not satisfy $Ly = 0$</p>

(3) Formulas for Trigonometric and Hyperbolic Functions

(1) $\cos(x) =$	$\frac{e^{ix} + e^{-ix}}{2}$
(2) $\sin(x) =$	$\frac{e^{ix} - e^{-ix}}{i2}$
(3) $\cosh(x) =$	$\frac{e^x + e^{-x}}{2}$
(4) $\sinh(x) =$	$\frac{e^x - e^{-x}}{2}$
(5) $\cos(a+b) =$	$\cos(a)\cos(b) - \sin(a)\sin(b)$

(6) $\sin(a+b) =$	$\sin(a)\cos(b) + \cos(a)\sin(b)$
(7) $\sinh(0) =$	0
(8) $\cosh(0) =$	1
(9) $\frac{d}{dx} \cosh x =$	$\sinh x, \quad \left. \frac{d}{dx} \cosh x \right _{x=0} = 0$
(10) $\frac{d}{dx} \sinh x =$	$\cosh x, \quad \left. \frac{d}{dx} \sinh x \right _{x=0} = 1$

(4) Integrals

(1) $1/x$	$\ln x + c$
(2) $\cos(x)$	$\sin(x) + c$
(3) $\sin(x)$	$-\cos(x) + c$
(4) $\tan(x)$	$-\ln \cos(x) + c$
(5) $\cot(x)$	$\ln \sin(x) + c$
(6) a^x	$a^x/\ln(a) + c$
(7) $\frac{1}{x^2 + a^2}$	$\frac{1}{a} \tan^{-1} \frac{x}{a} + c$
(8) $1/\sqrt{a^2 - x^2}$	$\sin^{-1}(x/a) + c$
(9) $-1/\sqrt{a^2 - x^2}$	$\cos^{-1}(x/a) + c$
(10) $x e^{ax}$	$\frac{e^{ax}}{a} \left(x - \frac{1}{a} \right) + c$
(11) $x^2 e^{ax}$	$\frac{e^{ax}}{a} \left(x^2 - \frac{2x}{a} + \frac{2}{a^2} \right) + c$