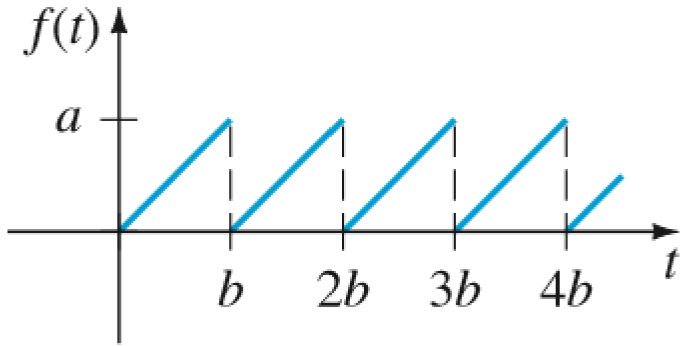
Homework 4 (Due: 1/5)

(1) Determine the Laplace transform of



sawtooth function

(2) Solve the following DE by the Laplace transform

$$y'' + 2y' = \delta(t-1), \quad y(0) = 0, y'(0) = 1$$

(3) Show that the following function set is orthogonal on the indicated interval.

$$\{\sin x, \sin 3x, \sin 5x, \ldots\}; \quad [0, \pi/2]$$

(4) Find the Fourier series of $f(x)$ on the given interval. Give the number to which the Fourier series converges at a point of discontinuity of $f(x)$.

$$f(x) = \begin{cases} \pi^2, & -\pi < x < 0 \\ \pi^2 - x^2, & 0 \le x < \pi \end{cases}$$

(5) Find the half-range cosine and sine expansions of the given function.

$$f(x) = \cos x, \quad 0 < x < \pi/2$$

(6) Proceed as in Example 4 in Section 11.3 to find the particular solution $x_p(t)$ of equation (11) when $m = 1$ and $k = 10$, and the driving force $f(t)$ is as given. Assume that when $f(t)$ is extended to the negative t-axis in a periodic manner, the resulting function is odd.

$$f(t) = 1 - t, \quad 0 < t < 2; \quad f(t + 2) = f(t)$$

(7) Represent the given function by an appropriate cosine or sine integral.

$$f(x) = e^{-|x|} \sin x$$

(8) Solve the following PDE by Fourier, Fourier cosine, or Fourier sine transform.

$$k\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} \qquad x > 0,\ t > 0$$

$$u(0, t) = 0,\ t > 0 \qquad u(x, 0) = \begin{cases} 1, & 0 < x < 1 \\ 0, & x > 1 \end{cases}$$

(9) Solve the following PDE by Fourier, Fourier cosine, or Fourier sine transform.

$$a\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} \qquad x > 0, t > 0$$

$$u(0, t) = 0, \quad t > 0$$

$$u(x, 0) = xe^{-x}, \quad \left.\frac{\partial u}{\partial t}\right|_{t=0} = 0, \quad x > 0$$

(10) Use separation of variables to solve the following PDE.

$$y\frac{\partial^2 u}{\partial x\,\partial y} + u = 0$$