Selected Topics in Engineering Mathematics Finals

(2 pages)

1. Solve the following nonlinear DE: (7 scores)

$$y'(x)y^2(x) = y''(x), \qquad y(0) = 3, \qquad y'(0) = 9$$

2. Solve the following PDEs: (27 scores)

(a) $2\frac{\partial}{\partial x}u(x,y) + \frac{\partial}{\partial y}u(x,y) = \sin(x) + \cos(y)$

(b) $\frac{\partial^2 u(x,y,z,t)}{\partial x^2} + \frac{\partial^2 u(x,y,z,t)}{\partial y^2} + \frac{\partial^2 u(x,y,z,t)}{\partial z^2} = \frac{\partial u(x,y,z,t)}{\partial t}$,

$0 < x < 1, \quad 0 < y < 1, \quad 0 < z < 1, \quad t > 0, \qquad u(0,y,z,t) = u(1,y,z,t) = 0$,

$u(x,0,z,t) = u(x,1,z,t) = 0, \qquad u(x,y,0,t) = u(x,y,1,t) = 0$.

(c) $\frac{\partial^2}{\partial x^2}u(x,y) + 4\frac{\partial^2}{\partial y^2}u(x,y) = 0, \ 0 < x < 6, \quad 0 < y < 6$

$u(0,y) = u(6,y) = 0, \qquad u(x,0) = x(6-x), \qquad \left.\frac{\partial}{\partial y}u(x,y)\right|_{y=0} = 0$

3. Suppose that (8 scores)

$$y(x) = 1 + \sin(\pi x), \qquad 0 \le x \le 2.$$

Approximate $y(x)$ by $c_0 + c_1x + c_2x^2$ such that $\int_0^2 \left(y(x) - c_0 - c_1x - c_2x^2\right)^2 dx$ is minimal.

4. Determine the following convolutions: (10 scores)

(a) $\delta(x) * \delta(2x) * \delta(3x) * \delta(x-1)$

(b) $\mathrm{sinc}(x/4) * \mathrm{sinc}(x/8) * \mathrm{sinc}(x/12) * \left(\cos(2\pi x)\sin(4\pi x)\right)$

5. Determine the Fourier transforms of the following functions. (18 scores)

(a) $g(x) = \mathrm{sinc}(4x)\cos(6\pi x)$

(Continued)

(b) $g(x) = x\exp(-2x^2)$

(c) $\sum_{n=-\infty}^{\infty} \delta(x-(2n+1))$

6. Suppose that (18 scores)

$$\mathbf{A} = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}.$$

(a) Determine det($\mathbf{A}\otimes\mathbf{B}$) where $\mathbf{B} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$.

(b) Determine the Jordan-canonical form of **A**.

(c) Determine $\lim_{\alpha\to 0}\left(\|\mathbf{A}\|_\alpha\right)^\alpha$, $\|\mathbf{A}\|_1, \|\mathbf{A}\|_2$, and $\|\mathbf{A}\|_\infty$ (Using the entry-wise matrix norm).

7. Suppose that (12 scores)

$$\mathbf{A} = \begin{bmatrix} 2 & 1 & 2 \\ 2 & -1 & 2 \end{bmatrix}$$

(a) Determine the SVD of **A**.

(b) Determine the generalized inverse of **A**.