

Homework 1 (Due: March 15th)

(1) Find the solutions of the following nonlinear DEs. (30 scores)

$$(a) \frac{y''(x)}{(y'(x))^2} = (1 + x^{-2}) \exp(-x^2 / 2) \quad y'(1) = \exp(1 / 2)$$

$$(hint): \int (1 + x^{-2}) \exp(-x^2 / 2) dx = -\frac{\exp(-x^2 / 2)}{x}$$

$$(b) (y'(x))^2 y''(x) = -\sin^2 x \cos x \quad y'(0) = 0$$

$$(c) y''(x) = 2y(x)y'(x) \quad y'(0) = y(0) = 1$$

(2) Solve the following PDEs. (30 scores)

$$(a) \sin y \frac{\partial u(x, y)}{\partial x} = \cos x \frac{\partial u(x, y)}{\partial y}$$

$$(b) 4 \frac{\partial^2 u(x, t)}{\partial x^2} = \frac{\partial u(x, t)}{\partial t}, \quad 0 < x < 3, \quad t > 0, \quad u(x, 0) = \sin^2(\pi x)$$

$$\frac{\partial}{\partial x} u(x, t) \Big|_{x=0} = \frac{\partial}{\partial x} u(x, t) \Big|_{x=3} = 0$$

(Conti.)

$$(c) \quad \frac{\partial^2 u(x, y)}{\partial x^2} + \frac{\partial^2 u(x, y)}{\partial y^2} = 0, \quad 0 < x < 2, \quad 0 < y < 2$$

$$u(0, y) = u(2, y) = u(x, 0) = 0, \quad u(x, 2) = \begin{cases} x & 0 < x \leq 1 \\ 2 - x & 1 < x < 2 \end{cases}$$

(3) Solve the following nonhomogeneous PDE. (10 scores)

$$x \frac{\partial u(x, y)}{\partial x} + y \frac{\partial u(x, y)}{\partial y} + x^2 + y^2 = 0$$

(4) Solve the following 1st order nonlinear DE numerically. The Matlab (or Python) code should also be handed out. (30 scores)

$$\frac{\partial y(x)}{\partial x} = \sqrt{xy} \cos(xy), \quad y(0) = 2, \quad 0 \leq x \leq 10, \quad x_{n+1} - x_n = 0.025$$

(a) By Euler's method.

(b) By modified Euler's method.

(c) By the RK4 method.