Homework 2 (Due: April 12th)

(1) Solve the following PDEs. (40 scores)

(a) $x\dfrac{\partial u(x,y,z)}{\partial x}+\left(y+1\right)\dfrac{\partial u(x,y,z)}{\partial y}+\left(z+2\right)\dfrac{\partial u(x,y,z)}{\partial z}=0$

(b) $\dfrac{\partial^2 u(x,y,t)}{\partial x^2}+\dfrac{\partial^2 u(x,y,t)}{\partial y^2}=\dfrac{\partial^2 u(x,y,t)}{\partial t^2}$ $\quad 0\le x\le 1,\quad 0\le y\le 1,\quad t\ge 0$

$u(0,y,t)=u(1,y,t)=u(x,0,t)=u(x,1,t)=0$

$u(x,y,0)=\sin\left(\pi x\right)\sin\left(\pi y\right)\qquad \left.\dfrac{\partial}{\partial t}u(x,y,t)\right|_{t=0}=0$

(c) $\dfrac{\partial^2 u}{\partial r^2}+\dfrac{1}{r}\dfrac{\partial u}{\partial r}+\dfrac{1}{r^2}\dfrac{\partial^2 u}{\partial \theta^2}=0$ $\qquad 0\le r\le 1\qquad 0\le\theta\le\pi/2$

$u\left(r,0\right)=u\left(r,\pi/2\right)=0,\quad u\left(1,\theta\right)=1$

(d) $\dfrac{\partial^2 u}{\partial r^2}+\dfrac{1}{r}\dfrac{\partial u}{\partial r}+\dfrac{\partial^2 u}{\partial z^2}=1$ $\qquad u(r,0)=u(r,5)=0,\quad 0<z<5,\quad 0<r<1,$

$u(0,z)=f\left(z\right)$

(Express the solution in terms of $f(z)$).

(2) Solve the following PDE by the 1-sided Laplace transform.

(10 scores)

$$\frac{\partial^2 u(x,y)}{\partial x^2} + 4\frac{\partial^2 u(x,y)}{\partial y^2} = 0, \quad 0 \le x \le 1, \quad y > 0$$

$$u(0,y) = -\left.\frac{\partial u}{\partial x}\right|_{x=0} = 2\sin(y/2), \quad u(x,0) = 0, \quad \left.\frac{\partial u}{\partial y}\right|_{y=0} = \exp(-x)$$

(3) Solve the following PDE by the Fourier integral.

(10 scores)

$$9\frac{\partial u(x,t)}{\partial x} = \frac{\partial u(x,t)}{\partial t} \qquad -\infty < x < \infty, \quad t > 0$$

$$u(x,0) = \cos(x) \qquad \text{(Hint): } \Im\{g(x-k)\} = e^{-j2\pi kf}G(f)$$

(4) (a) Convert 1 and x into an orthonormal function set for $x \in [0, 4]$.

(b) Expand $g(x) = 2\text{-}x$ for $x < 2$ and $g(x) = 0$ otherwise by $q(x) = c_0 + c_1 x$

such that $\int_0^4 \left(g(x) - q(x)\right)^2 dx$ is minimized. (20 scores)

(5) Suppose that there is a set of six 'discrete' basis.

$$b_k[n] = n(n-1)\cdots(n-k) \qquad n = 0, 1, 2, \cdots, 12$$

$$k = 0, 1, 2, 3, 4, 5$$

(a) Use the Gram-Schmidt method (written by Matlab or Python) to convert $b_k[n]$ ($k = 0\sim5$) into an orthonormal basis set.

(b) Use the Gram-Schmidt method (written by Matlab or Python) to convert $b_k[n]$ ($k = 0\sim5$) into an orthonormal basis set if the weight is $w[n] = \exp(-n)$.

The codes should be handed out by NTUCool. (20 scores)