

Homework 3 (Due: May 3rd)

(1) Suppose that

$$G(f) = \int_{-\infty}^{\infty} g(x) e^{-j2\pi fx} dx, \quad G_1(\omega) = \int_{-\infty}^{\infty} g(x) e^{-j\omega x} dx$$

$$G_2(\omega) = \sqrt{\frac{1}{2\pi}} \int_{-\infty}^{\infty} g(x) e^{-j\omega x} dx \quad G_3(\alpha) = \int_{-\infty}^{\infty} g(x) e^{j\alpha x} dx$$

Try to express $G_1(\omega), G_2(\omega), G_3(\alpha)$ in terms of $G(f)$. (10 scores)

(2) Determine the Fourier transform of the following functions.

(a) $g(x) = \exp(-2(x+1))$ for $x > -1$, $g(x) = 0$ for $x < -1$.

(b) $g(x) = \sin(6\pi x)$ for $0 < x < 6$, $g(x) = 0$ otherwise.

(c) $g(x) = (x+1)\exp(-x^2)$. (30 scores)

(3) Determine

$$\int_{-\infty}^{\infty} |\text{sinc}(2x) \cos \pi x|^2 dx \quad (10 \text{ scores})$$

(4) Determine the following convolutions.

(a) $\cos(4\pi x)\sin(6\pi x) * \text{sinc}(8x) * \text{sinc}(6x) * \text{sinc}(4x)$

(b) $\exp(-\pi x^2) * \cos(4\pi x) * \delta'(x)$ (20 scores)

(5) Determine

$$G(f, h) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-j2\pi fx} e^{-j2\pi hy} g(x, y) dx dy$$

$$\text{where } g(x, y) = \begin{cases} 1 & \text{for } \sqrt{(x/2)^2 + 4y^2} < 1 \\ 0 & \text{otherwise} \end{cases} \quad (10 \text{ scores})$$

(6) Using a Matlab or Python code to determine the continuous Fourier transform of the following functions by the DFT.

(a) $g(x) = 4 - x^2$ for $0 < x < 2$, $g(x) = 0$ otherwise, $\Delta_x = 0.05$.

(b) $g(x) = \text{sinc}(x^2)$ for $-2 < x < 2$, $g(x) = 0$ otherwise, $\Delta_x = 0.1$.

The codes should be handed out by NTUCool. (20 scores)