Homework 4 (Due: May 24th)

(1) Suppose that $x[n] = 1$ for $n = 0, 4, 8, 12, 16$, and $x[n] = 0$ otherwise.

$y[n] = 1$ for $n = 0, 10$, and $y[n] = 0$ otherwise.

Determine IDFT{DFT($x[n]$)DFT($y[n]$)} where the number of points of the DFTs and the inverse DFT (IDFT) are all 20. (10 scores)

(2) Suppose that

$$\mathbf{A} = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix} \qquad \mathbf{A}_2 = \mathbf{A} \otimes \mathbf{A} \qquad \mathbf{A}_n = \mathbf{A} \otimes \mathbf{A}_{n-1}$$

$n = 3, 4, 5, \ldots..$

(a) Determine the eigenvectors and eigenvalues of $\mathbf{A_2}$.

(b) Determine $\lim_{\alpha \to 0} \left(\|\mathbf{A}\|_\alpha \right)^\alpha$, $\|\mathbf{A}\|_1$, $\|\mathbf{A}\|_2$, and $\|\mathbf{A}\|_\infty$.

(c) Determine $\lim_{\alpha \to 0} \left(\|\mathbf{A}_n\|_\alpha \right)^\alpha$, $\|\mathbf{A}_n\|_1$, $\|\mathbf{A}_n\|_2$, and $\|\mathbf{A}_n\|_\infty$.

Express the solution in terms of n. (30 scores)

(3) Suppose that

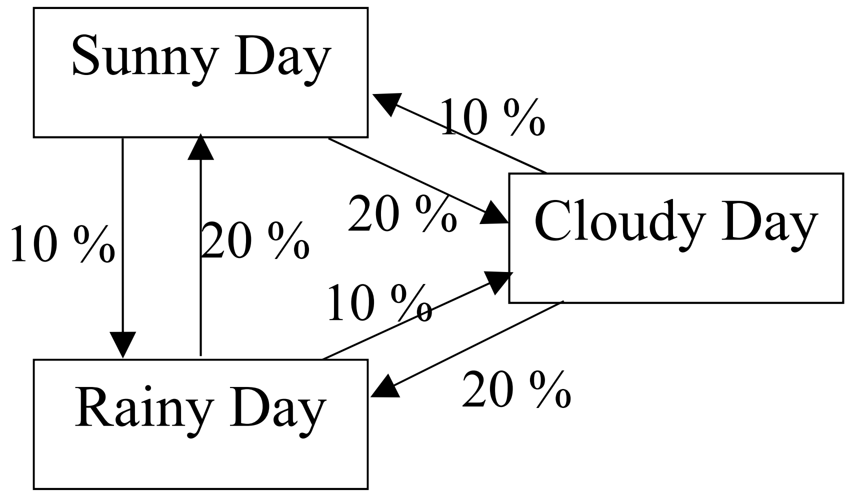
$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) Determine the Jordan-canonical form of $\mathbf{A}$.

(b) Determine $\sin(\mathbf{A})$. (20 scores).

(4) Suppose that if the weather today is sunny, then the probabilities that the next day is cloudy and rainy are 20% and 10%, respectively. If today is cloudy, then the probabilities that the next day is sunny and rainy are 10% and 20%, respectively. If today is rainy, then the probabilities that the next day is sunny and cloudy are 20% and 10%, respectively.

(a) If today is sunny, predict the probability that the weather is sunny 3 days later.
(b) Also, predict the probability that the weather is sunny infinite number of days later. (20 scores)



(5) Suppose that $\mathbf{y}$ = [5, 3, 5, 3, 5, 6, 6, 5];

$$\mathbf{b}_m[n] = [1 \quad \cos m \quad \cos 2m \quad \cos 3m \quad \cos 4m \quad \cos 5m \quad \cos 6m \quad \cos 7m]$$

Use a Matlab or Python code to

(a) Find c_0 and c_1 to minimize

$$\left\| \mathbf{y} - c_0\mathbf{b_0} - c_1\mathbf{b_1} \right\|_2$$

(b) Find c_0, c_1, c_2 , and c_3 to minimize

$$\left\| \mathbf{y} - c_0\mathbf{b_0} - c_1\mathbf{b_1} - c_2\mathbf{b_2} - c_3\mathbf{b_3} \right\|_2$$

(20 scores)

The code should be handed out by NTUCool.