Homework 5 (Due: June 21st)

(1) Determine the generalized inverse of the following matrices.

(a) $\mathbf{A} = \begin{bmatrix} 1 & -1 & -1 \\ -1 & 4 & 2 \\ -1 & 2 & 4 \end{bmatrix}$, (b) $\mathbf{B} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 2 \end{bmatrix}$ (20 scores)

(2) Perform the SVD for the following matrices

(a) $\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$, (b) $\mathbf{B} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$ (20 scores)

(3) Suppose that the PDF of X is:

$$f_X(x)=\begin{cases}\frac{3}{4}\left(1-x^2\right) & for\ -1<x<1\\ 0 & otherwise\end{cases}$$

(a) Determine the standard deviation, the skewness, and the kurtosis of X.

(b) Suppose that $Y = 2X+1$. Determine the standard deviation, the skewness, and the kurtosis of Y. (20 scores)

(4) Determine the correlation of X and Y if

(a) $$f_{X,Y}(x,y)=\begin{cases}\frac{1}{12} & if\ 1<\max\left(|x|,|y|\right)<2\\ 0 & otherwise\end{cases}$$

(b) $$f_{X,Y}(x,y)=\begin{cases}\frac{1}{50} & if\ 0<x<10\ and\ 0<y<x\\ 0 & otherwise\end{cases}$$

(20 scores)

(5) Suppose that

$$P_X(n) = \frac{1}{2N+1} \qquad \text{for } n = -N, -N+1, \ldots, N,$$

$$P_X(n) = 0 \qquad \text{otherwise,}$$

$$P_Y(n) = \tanh\left(\frac{\sigma}{2}\right)\exp\left(-\sigma|n|\right) \qquad \text{for all integer } n.$$

Use a Matlab or Python program to find σ (σ is constrained to be a multiple of 0.01, $0 < \sigma \leq 1$) such that the KL divergence from $P_Y(n)$ to $P_X(n)$ is minimum in the cases where

(a) $N = 10$ and (b) $N = 20$.

The code should be handed out by NTUCool. (20 scores)