X. Other Applications of Time-Frequency Analysis

Applications

(1) Finding Instantaneous Frequency

(2) Signal Decomposition

(3) Filter Design

(4) Sampling Theory

(5) Modulation and Multiplexing

(6) Electromagnetic Wave Propagation

(7) Optics

(8) Radar System Analysis

(9) Random Process Analysis

(10) Music Signal Analysis

(11) Biomedical Engineering

(12) Accelerometer Signal Analysis

(13) Acoustics

(14) Data Compression

(15) Spread Spectrum Analysis

(16) System Modeling

(17) Image Processing

(18) Economic Data Analysis

(19) Signal Representation

(20) Seismology

(21) Geology

(22) Astronomy

(23) Oceanography

10-1 Sampling Theory

Number of sampling points == Sum of areas of time frequency distributions
+ the number of extra parameters

- How to make the area of time-frequency smaller?

(1) Divide into several components.

(2) Use chirp multiplications, chirp convolutions, fractional Fourier transforms, or linear canonical transforms to reduce the area.

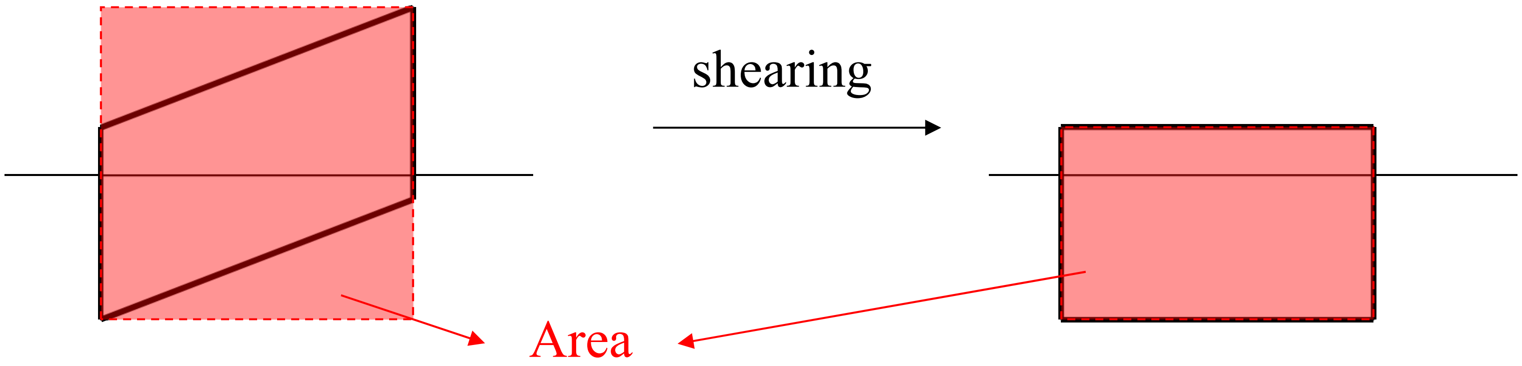
[Ref] X. G. Xia, "On bandlimited signals with fractional Fourier transform," *IEEE Signal Processing Letters*, vol. 3, no. 3, pp. 72-74, March 1996.

[Ref] J. J. Ding, S. C. Pei, and T. Y. Ko, "Higher order modulation and the efficient sampling algorithm for time variant signal," *European Signal Processing Conference*, pp. 2143-2147, Bucharest, Romania, Aug. 2012.

Analytic Signal Conversion

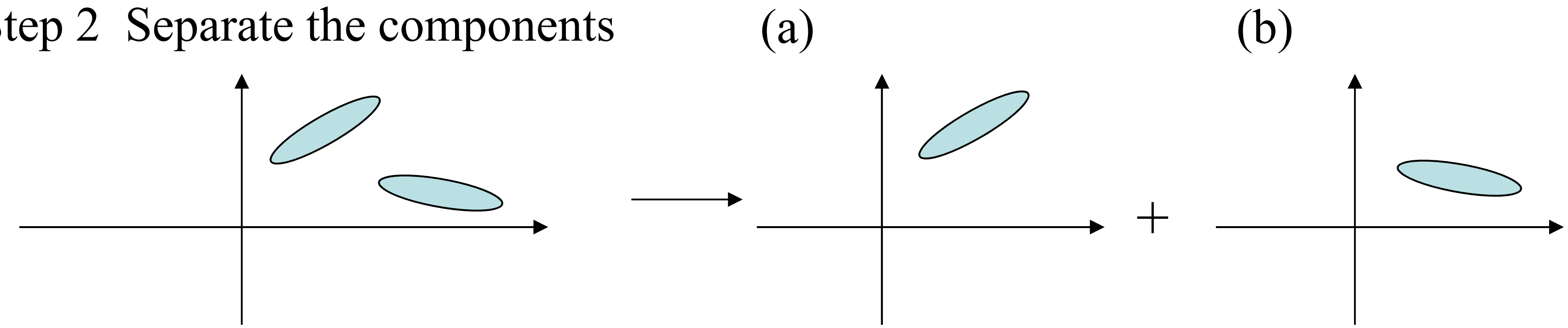
$$x(t) \rightarrow x_a(t) = x(t) + jx_H(t)$$

Shearing



Step 1 Analytic Signal Conversion

Step 2 Separate the components



Step 3 Use shearing or rotation to minimize the "area" to each component

Step 4 Use the conventional sampling theory to sample each components

傳統的取樣方式

$$x_d[n] = x(n\Delta_t) \qquad \Delta_t < 1/F$$

重建： $$x(t) = \sum_n x_d[n]\operatorname{sinc}\left(\frac{t}{\Delta_t} - n\right)$$

新的取樣方式 $x_H(t)$: Hilbert transform of $x(t)$

(1) $x(t) \to x_a(t) = x(t) + jx_H(t)$

(2) $x_a(t) \to x_a(t) = x_1(t) + x_2(t) + \cdots + x_K(t)$

(3) $y_k(t) = \exp\left(j2\pi a_k t^2\right) x_k(t) \qquad k = 1, 2, \ldots, K$

(4) $x_{d,k}[n] = y_k(n\Delta_{t,k}) \qquad k = 1, 2, \ldots, K$

$$= \exp\left(j2\pi a_k n^2 \Delta_{t,k}^2\right) x_k(n\Delta_{t,k})$$

重建：

(1) $y_k(t) = \sum_n x_{d,k}[n] \operatorname{sinc}\left(\frac{t}{\Delta_{t,k}} - n\right)$

(2) $x_k(t) = \exp\left(-j2\pi a_k t^2\right) y_k(t)$

(3) $x_a(t) = x_1(t) + x_2(t) + \cdots + x_K(t)$

(4) $x(t) = \mathcal{Re}\left\{x_a(t)\right\}$

嚴格來說，沒有一個信號的 時頻分佈的「面積」是有限的。

Theorem:

If $x(t)$ is time limited ($x(t) = 0$ for $t < t_1$ and $t > t_2$)

then it is impossible to be frequency limited

If $x(t)$ is frequency limited ($X(f) = 0$ for $f < f_1$ and $f > f_2$)

then it is impossible to be time limited

但是我們可以選一個 "threshold" Δ

時頻分析 $|X(t, f)| > \Delta$ 或 的區域的面積是有限的

實際上，以「面積」來討論取樣點數，是犧牲了一些精確度。

只取 $t \in [t_1, t_2]$ and $f \in [f_1, f_2]$ 犠牲的能量所佔的比例

$$err = \frac{\int_{-\infty}^{t_1} |x(t)|^2 dt + \int_{t_2}^{\infty} |x(t)|^2 dt + \int_{-\infty}^{f_1} |X_1(f)|^2 df + \int_{f_2}^{\infty} |X_1(f)|^2 df}{\int_{-\infty}^{\infty} |x(t)|^2 dt}$$

$X_1(f) = FT[x_1(t)]$, $x_1(t) = x(t)$ for $t \in [t_1, t_2]$, $x_1(t) =$ 0 otherwise

- For the Wigner distribution function (WDF)

$$|x(t)|^2 = \int_{-\infty}^{\infty} W_x(t, f)\, df, \qquad |X(f)|^2 = \int_{-\infty}^{\infty} W_x(t, f)\, dt$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} W_x(t, f)\, df dt = \int_{-\infty}^{\infty} |x(t)|^2 dt \quad = \quad \text{energy of } x(t).$$

$$|x(t)|^2 = \int_{-\infty}^{\infty} W_x(t,f)\,df \qquad |X(f)|^2 = \int_{-\infty}^{\infty} W_x(t,f)\,dt$$

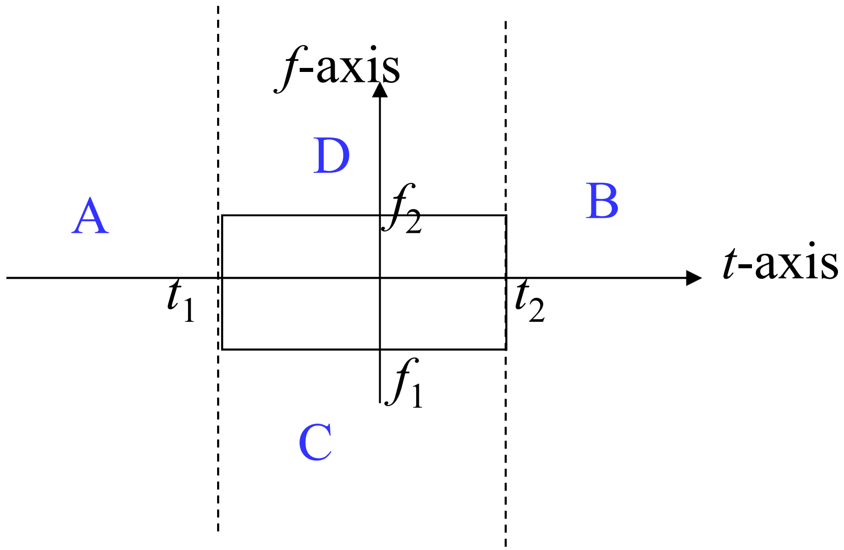
$$\int_{-\infty}^{t_1} |x(t)|^2\,dt + \int_{t_2}^{\infty} |x(t)|^2\,dt + \int_{-\infty}^{f_1} |X_1(f)|^2\,df + \int_{f_2}^{\infty} |X_1(f)|^2\,df$$

$$= \int_{-\infty}^{t_1}\int_{-\infty}^{\infty} W_x(t,f)\,dfdt + \int_{t_2}^{\infty}\int_{-\infty}^{\infty} W_x(t,f)\,dfdt + \int_{-\infty}^{\infty}\int_{-\infty}^{f_1} W_{x_1}(t,f)\,dfdt + \int_{-\infty}^{\infty}\int_{f_2}^{\infty} W_{x_1}(t,f)\,dfdt$$

$$= \int_{-\infty}^{t_1}\int_{-\infty}^{\infty} W_x(t,f)\,dfdt + \int_{t_2}^{\infty}\int_{-\infty}^{\infty} W_x(t,f)\,dfdt + \int_{t_1}^{t_2}\int_{-\infty}^{f_1} W_{x_1}(t,f)\,dfdt + \int_{t_1}^{t_2}\int_{f_2}^{\infty} W_{x_1}(t,f)\,dfdt$$

$$\cong \underbrace{\int_{-\infty}^{t_1}\int_{-\infty}^{\infty} W_x(t,f)\,dfdt}_{A} + \underbrace{\int_{t_2}^{\infty}\int_{-\infty}^{\infty} W_x(t,f)\,dfdt}_{B} + \underbrace{\int_{t_1}^{t_2}\int_{-\infty}^{f_1} W_x(t,f)\,dfdt}_{C} + \underbrace{\int_{t_1}^{t_2}\int_{f_2}^{\infty} W_x(t,f)\,dfdt}_{D}$$

$$err \cong 1 - \frac{\int_{t_1}^{t_2}\int_{f_1}^{f_2} W_x(t,f)\,dfdt}{\int_{-\infty}^{\infty} |x(t)|^2\,dt}$$



10-2 Modulation and Multiplexing

With the aid of

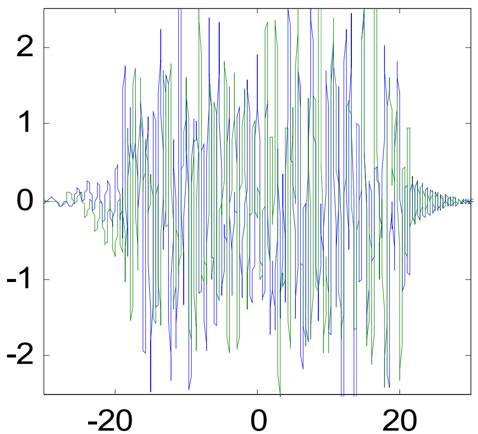
(1) the Gabor transform (or the Gabor-Wigner transform)

(2) horizontal and vertical shifting, dilation, shearing, generalized shearing, and rotation.

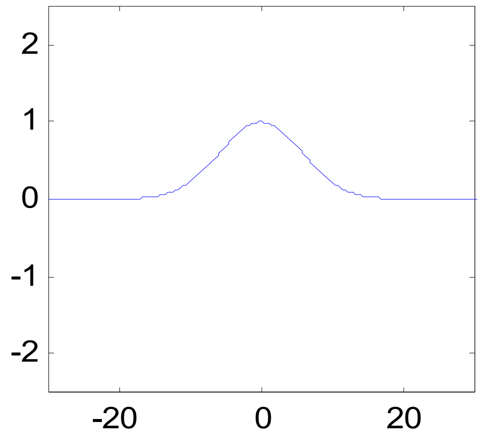
[Ref] C. Mendlovic and A. W. Lohmann, "Space-bandwidth product adaptation and its application to superresolution: fundamentals," *J. Opt. Soc. Am. A*, vol. 14, pp. 558-562, Mar. 1997.

[Ref] S. C. Pei and J. J. Ding, "Relations between Gabor transforms and fractional Fourier transforms and their applications for signal processing," vol. 55, issue 10, pp. 4839-4850, *IEEE Trans. Signal Processing*, 2007.

Example



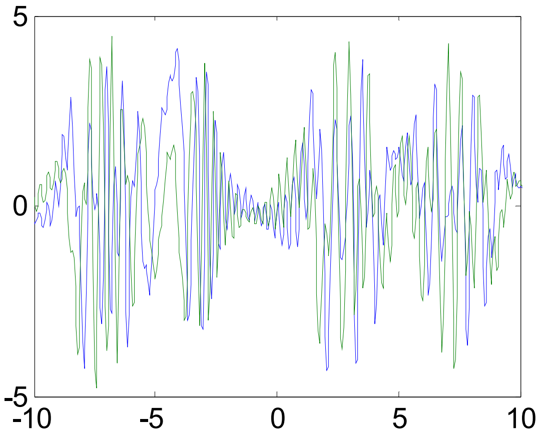
(a) G(u), consisted of 7 components



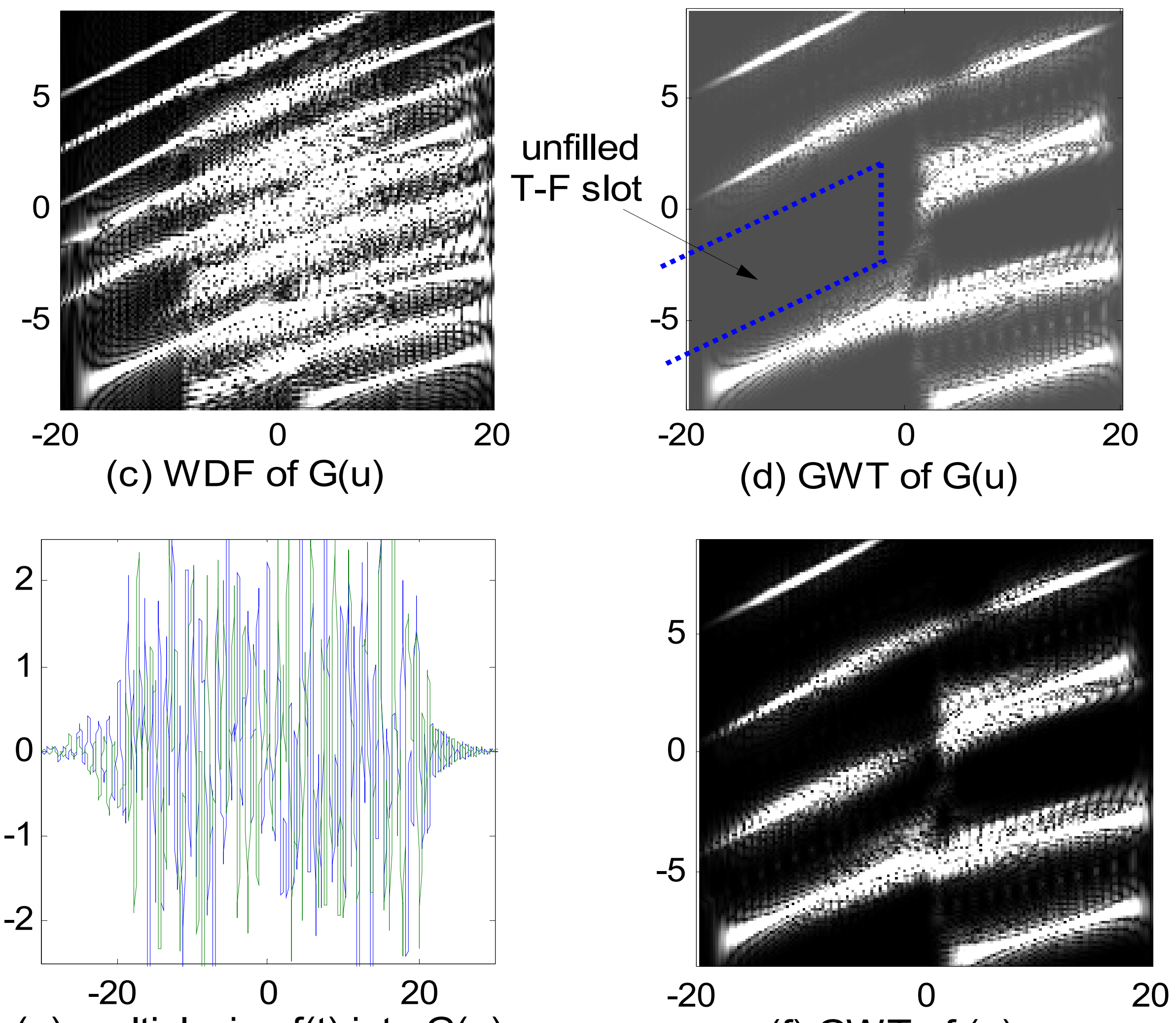
(b) f(t), the signal to be modulated

FT

We want to add $f(t)$ into $G(u)$



(no empty band)



(c) WDF of G(u)
(d) GWT of G(u)
(e) multiplexing f(t) into G(u)
(f) GWT of (e)

◎ Conventional Modulation Theory

The signals $x_1(t)$, $x_2(t)$, $x_3(t)$, ……., $x_K(t)$ can be transmitted successfully if

$$\text{Allowed Bandwidth} \geq \sum_{k=1}^{K} B_k$$

B_k: the bandwidth (including the negative frequency part) of $x_k(t)$

◎ Modulation Theory Based on Time-Frequency Analysis

The signals $x_1(t)$, $x_2(t)$, $x_3(t)$, ……., $x_K(t)$ can be transmitted successfully if

$$\text{Allowed Time duration} \times \text{Allowed Bandwidth} \geq \sum_{k=1}^{K} A_k$$

A_k: the area of the time-frequency distribution of $x_k(t)$

- The interference is inevitable.

How to estimate the interference?

10-3 Electromagnetic Wave Propagation

Time-Frequency analysis can be used for

Wireless Communication

Optical system analysis

Laser

Radar system analysis

Propagation through the free space (Fresnel transform): chirp convolution

Propagation through the lens or the radar disk: chirp multiplication

Fresnel Transform ：描述電磁波在空氣中的傳播 (See page 260-264)

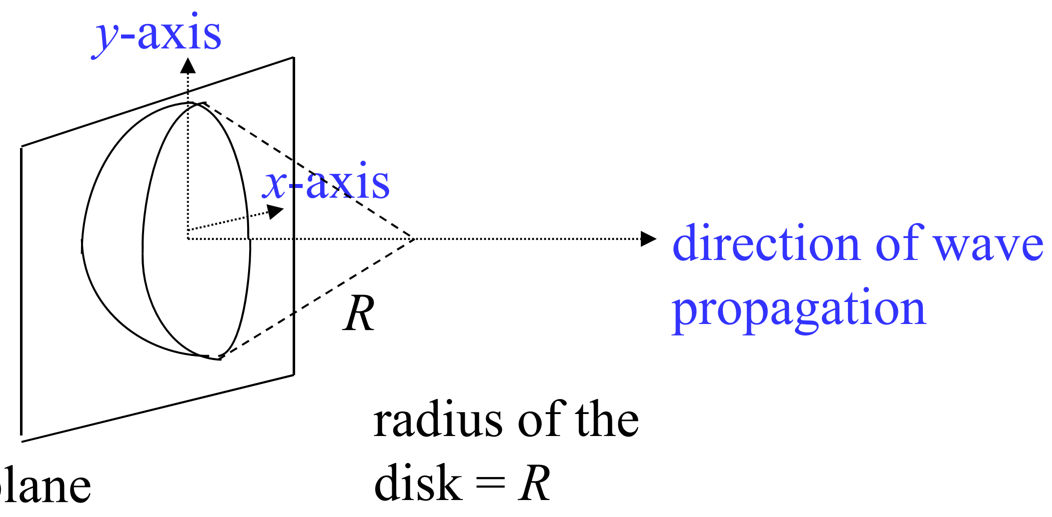
電磁波包括光波、雷達波、紅外線、紫外線.........

Fresnel transform == LCT with parameters $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & \lambda z \\ 0 & 1 \end{bmatrix}$

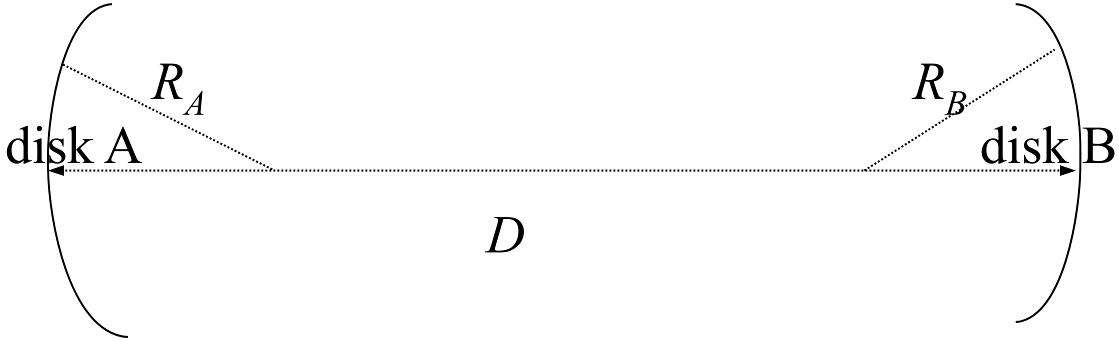
思考： (1) STFT 或 WDF 哪一個比較適合用在電磁波傳播的分析？

(2) 為何波長越短的電磁波，在空氣中散射的情形越少？

(4) Spherical Disk



Disk 相當於 LCT $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1/\lambda R & 1 \end{bmatrix}$ 的情形



相當於 LCT $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1/\lambda R_B & 1 \end{bmatrix} \begin{bmatrix} 1 & \lambda D \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1/\lambda R_A & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1 - D/R_A & -\lambda D \\ -\frac{1}{\lambda}\left(R_A^{-1} - R_B^{-1} + R_A^{-1} R_B^{-1} D\right) & 1 + D/R_B \end{bmatrix}$$

的情形

10-4 Music and Acoustic Signal Analysis

Music Signal Analysis

Acoustic

Voiceprint (Speaker) Recognition

Speech Signal ：

(1) 不同的人說話聲音頻譜不同 (聲紋 voiceprint)

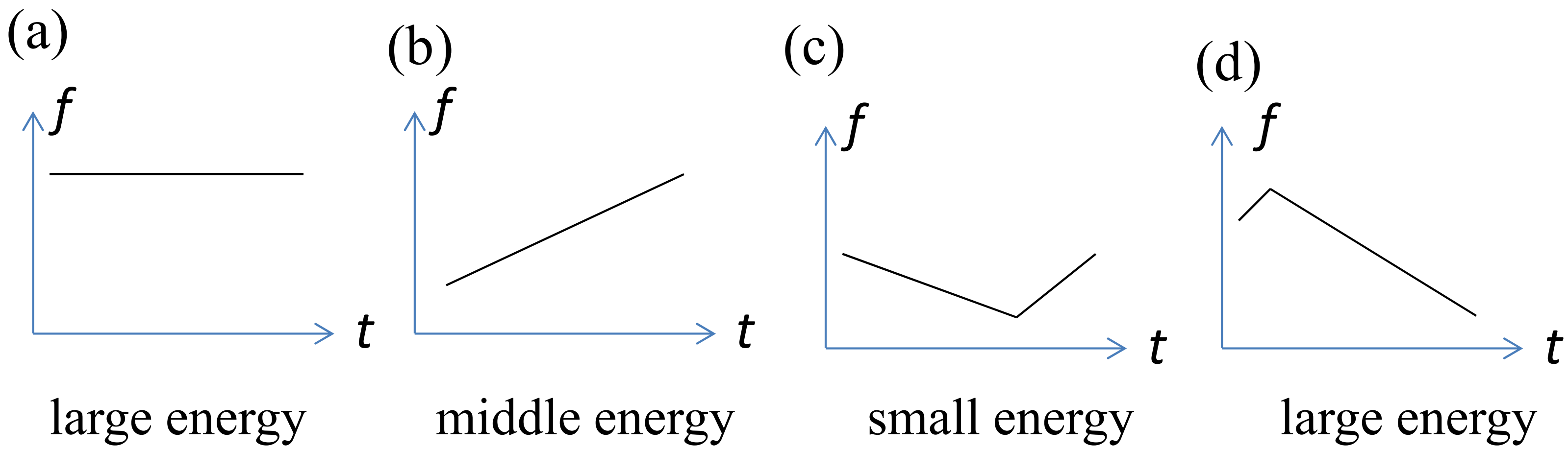
(2) 同一個人但不同的字音，頻譜不一樣

(3) 語調 (第一、二、三、四聲和輕聲) 不同，則頻譜變化的情形也不同

(4) 即使同一個字音，子音和母音的頻譜亦不相同

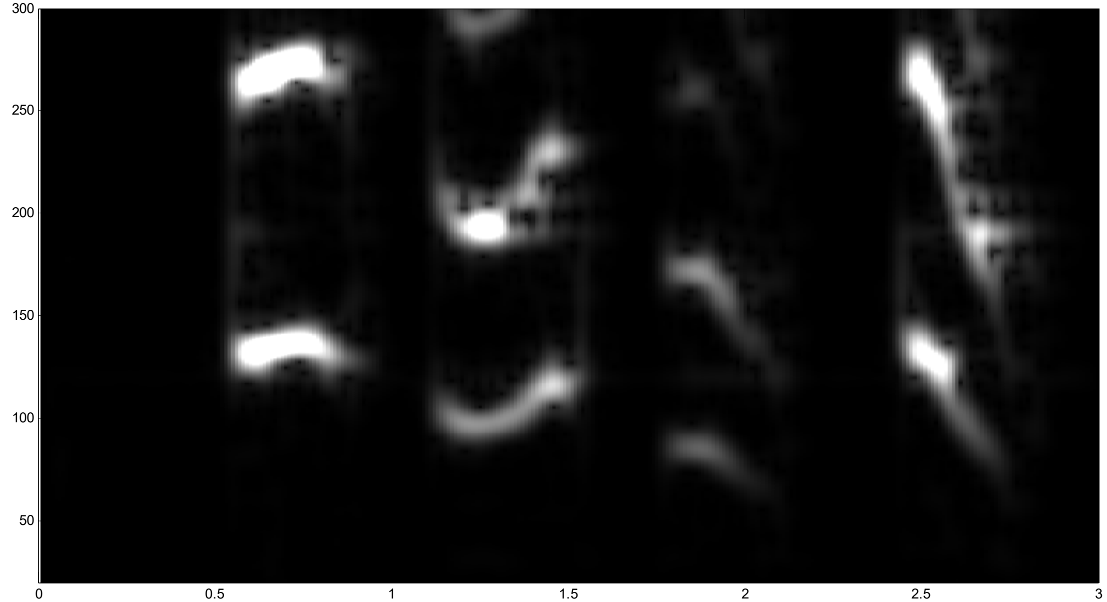
(5) 雙母音本身就會有頻譜的變化

- 王小川，“語音訊號處理”，第二章，全華出版，台北，民國94年。



Typical relations between time and the instantaneous frequencies for (a) the 1^{st} tone, (b) the 2^{nd} tone, (c) the 3^{rd} tone, and (d) the 4^{th} tone in Chinese.

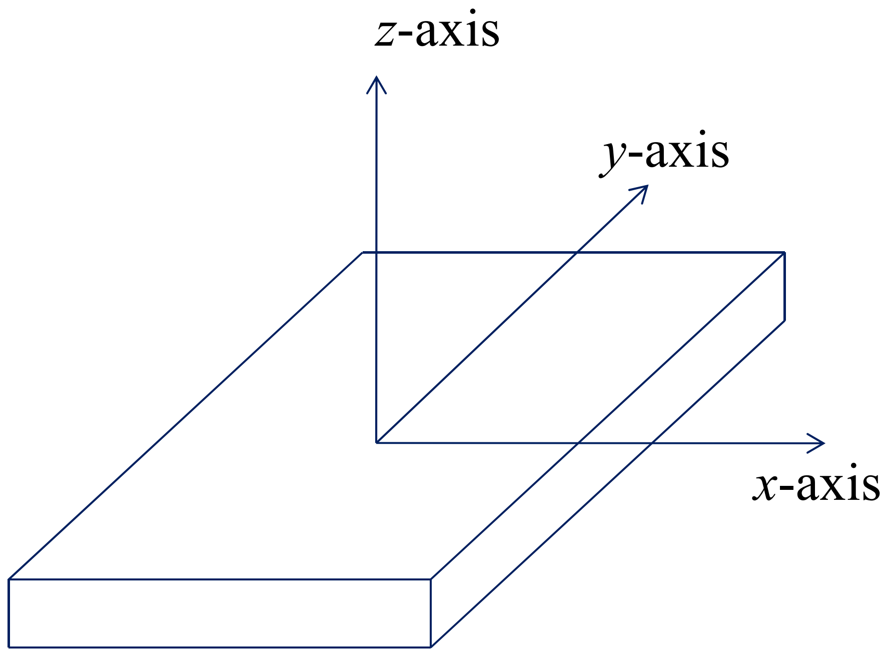
X. X. Chen, C. N. Cai, P. Guo, and Y. Sun, "A hidden Markov model applied to Chinese four-tone recognition," *ICASSP*, vol. 12, pp. 797-800, 1987.

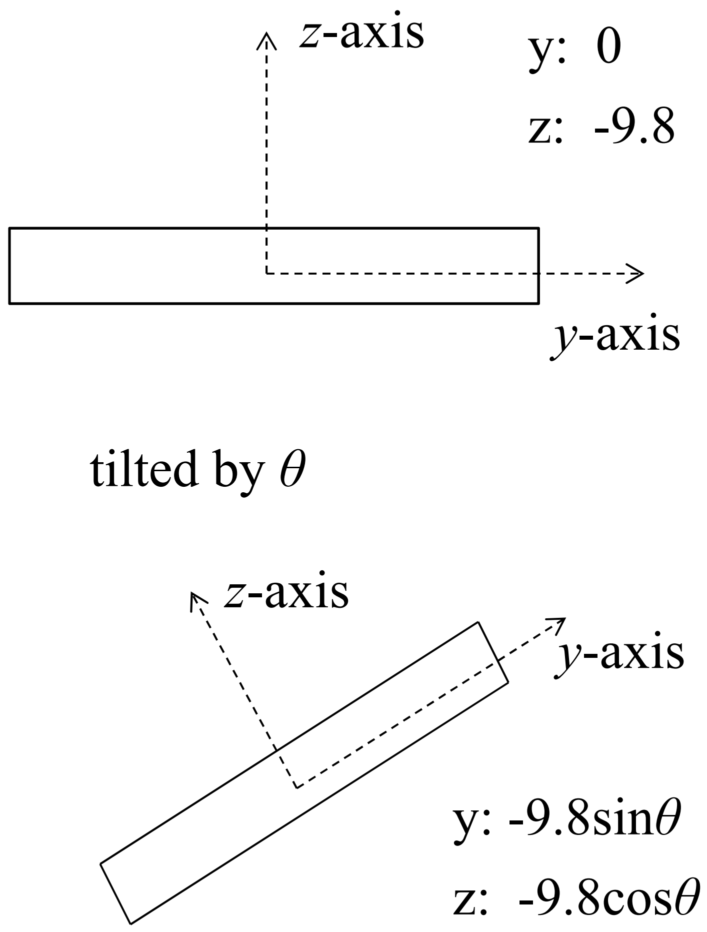


Y1, Y2, Y3, Y4

10-5 Accelerometer Signal Analysis

The 3-D Accelerometer (三軸加速規) can be used for identifying the activity of a person.



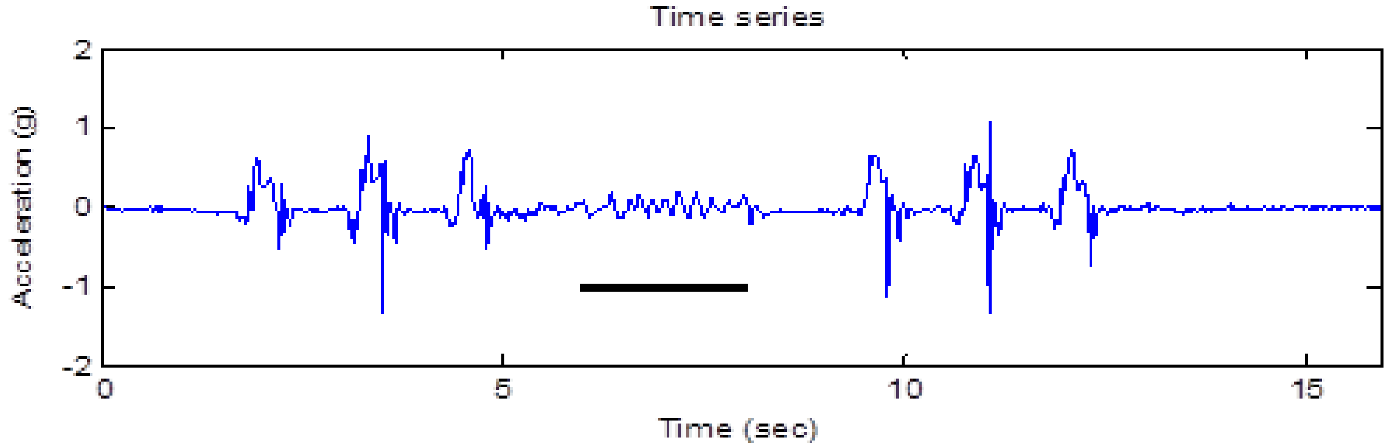


Using the 3D accelerometer + time-frequency analysis, one can analyze the activity of a person.

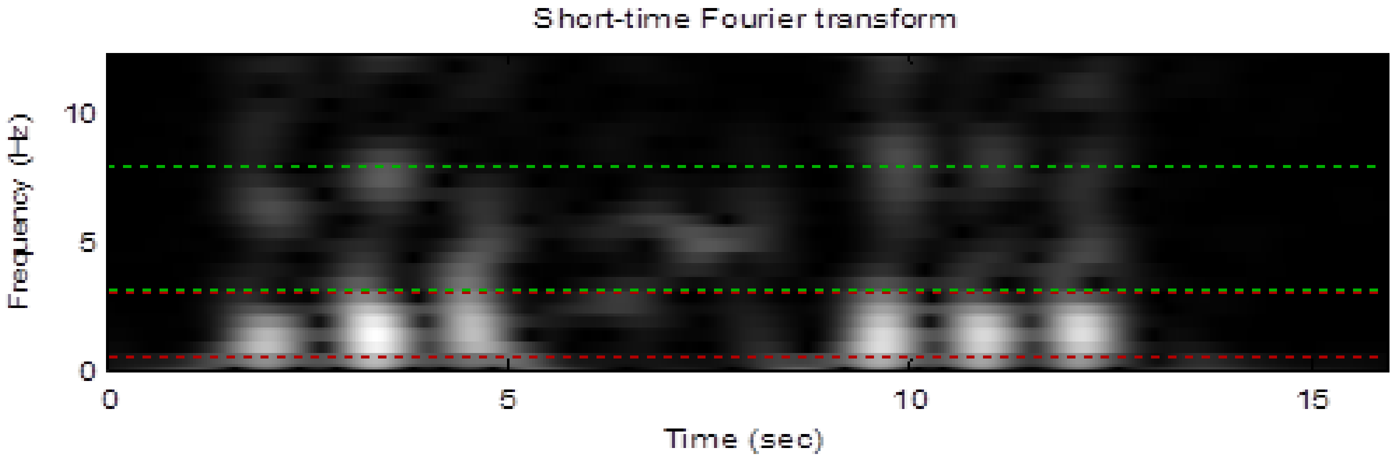
Walk, Run (Pedometer 計步器)

Healthcare for the person suffered from Parkinson's disease

3D accelerometer signal for a person suffering from Parkinson's disease



The result of the short-time Fourier transform



Y. F. Chang, J. J. Ding, H. Hu, Wen-Chieh Yang, and K. H. Lin, "A real-time detection algorithm for freezing of gait in Parkinson's disease," *IEEE International Symposium on Circuits and Systems*, Melbourne, Australia, pp. 1312-1315, May 2014

10-6 Other Applications

時頻分析適用於頻譜會隨著時間而改變的信號

Biomedical Engineering (心電圖 (ECG), 肌電圖 (EMG), 腦電圖, ……)

Communication and Spread Spectrum Analysis

Economic Data Analysis

Seismology

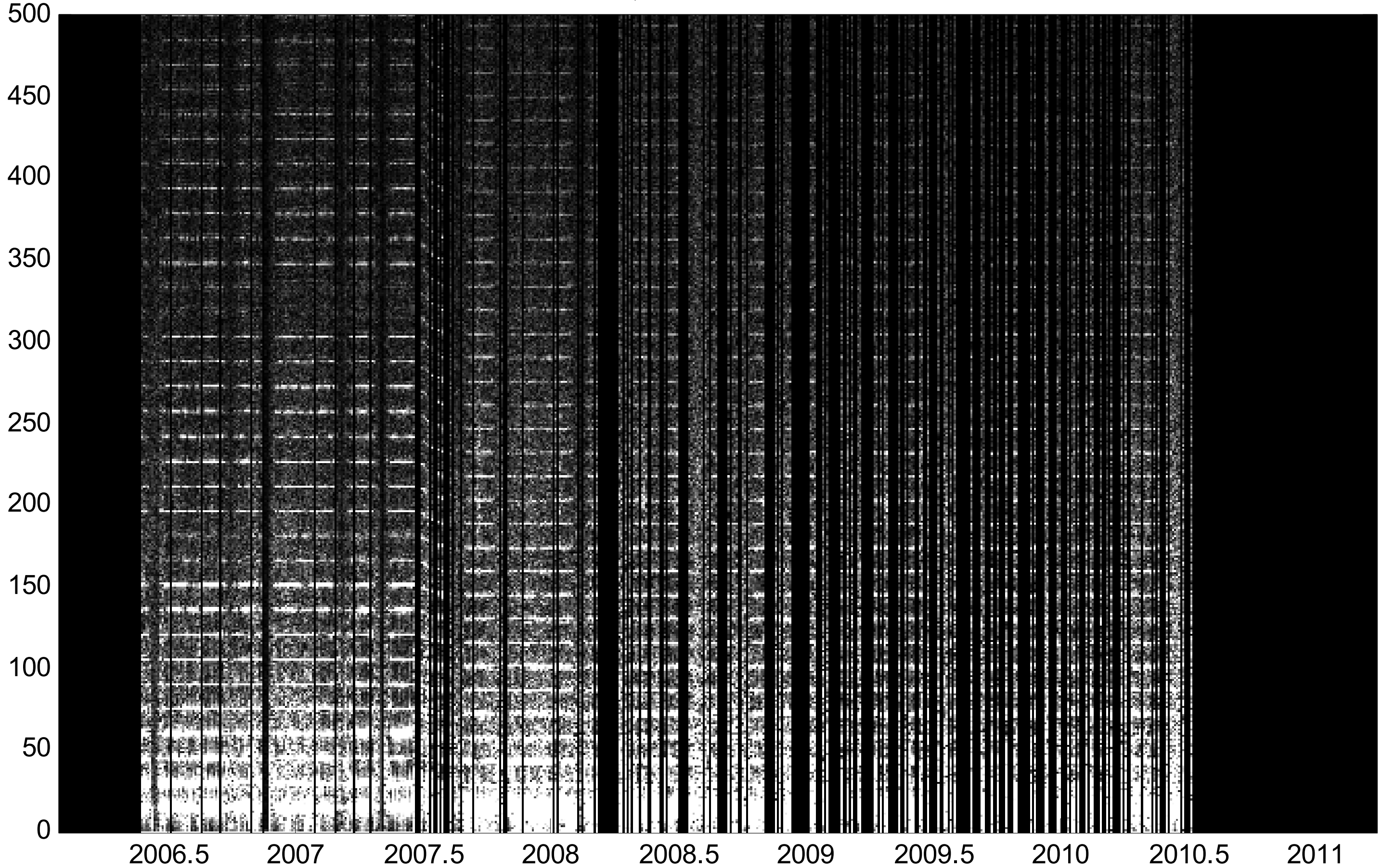
Geology

Astronomy

Oceanography

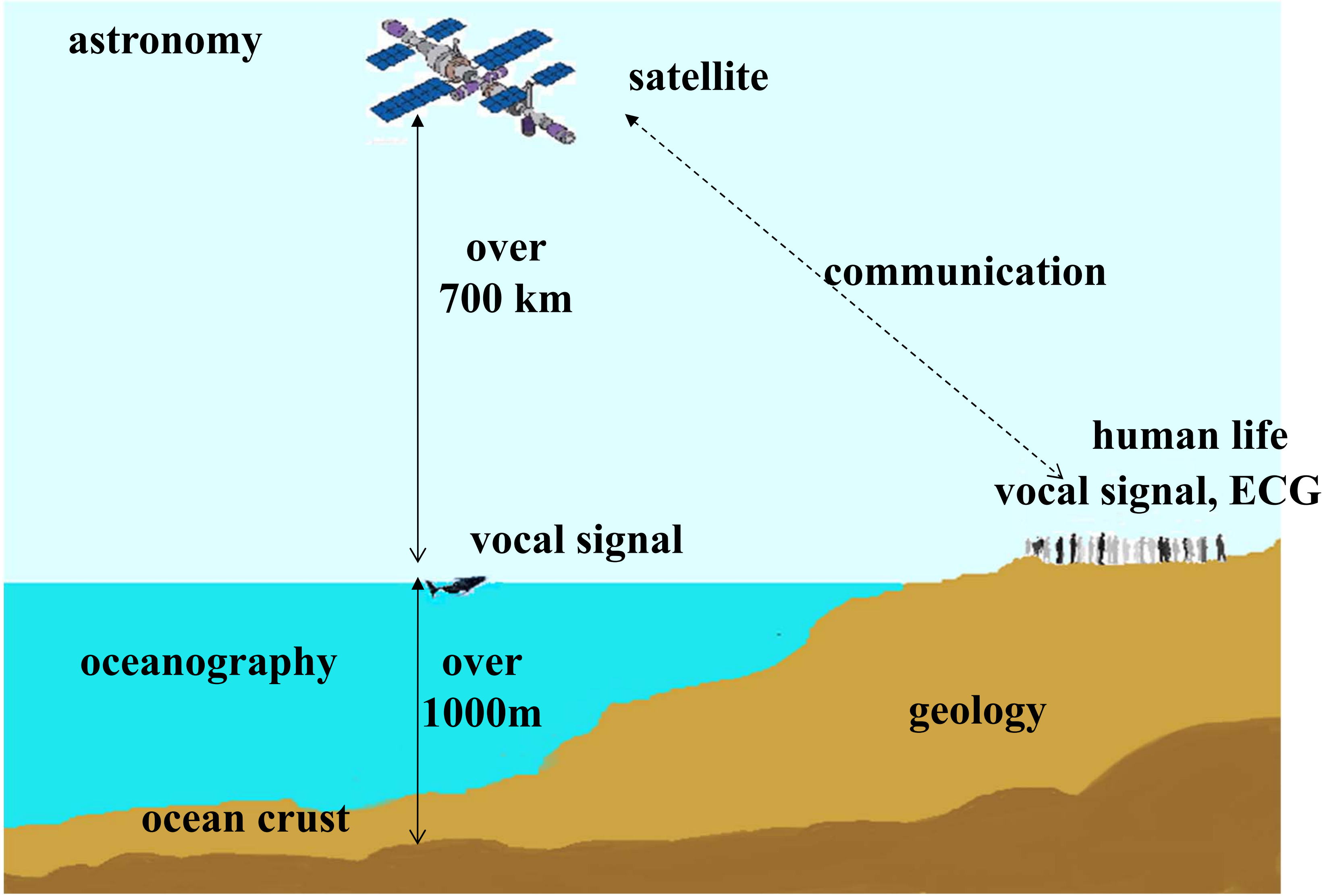
Satellite Signal

Short-time Fourier transform of the power signal from a satellite
福爾摩沙衛星三號



C. J. Fong, S. K. Yang, N. L. Yen, T. P. Lee, C. Y. Huang, H. F. Tsai, S. Wang, Y. Wang, and J. J. Ding, "Preliminary studies of the applications of HHT (Hilbert-Huang transform) on FORMOSAT-3/COSMIC GOX payload trending data," *6th FORMOSAT-3/COSMIC Data Users' Workshop*, Boulder, Colorado, USA, Oct. 2012

時頻分析的應用範圍


astronomy
satellite
over
700 km
communication
human life
vocal signal, ECG
vocal signal
oceanography
over
1000m
geology
ocean crust

附錄十：幾個常見的資料蒐尋方法

(1) Google 學術搜尋

http://scholar.google.com.tw/

(太重要了，不可以不知道) 只要任何的書籍或論文，在網路上有電子版，都可以用這個功能查得到

輸入關鍵字，或期刊名，或作者

再按「搜尋」，就可找到想要的資料

(2) 尋找 IEEE 的論文

http://ieeexplore.ieee.org/Xplore/guesthome.jsp

(3) Wikipedia

(4) Github (搜尋 code)

(5) 數學的百科網站

http://eqworld.ipmnet.ru/index.htm

有多個 tables，以及對數學定理的介紹

(6) 傳統方法：去圖書館找資料

台大圖書館首頁 http://www.lib.ntu.edu.tw/

或者去 http://www.lib.ntu.edu.tw/tulips

(7) 查詢其他圖書館有沒有我要找的書

「台大圖書館首頁」⟶「其他圖書館」

(8) 找尋電子書

「台大圖書館首頁」⟶「電子書」或「免費電子書」

(9) 查詢一個期刊是否為 SCI

Step 1: 先去 http://scientific.thomson.com/mjl/

Step 2: 在 Search Terms 輸入期刊全名

Search Type 選擇 “Full Journal Title”，再按 “Search”

Step 3: 如果有找到這期刊，那就代表這個期刊的確被收錄在 SCI

(10) 想要對一個東西作入門但較深入的了解:

看 journal papers 或 Wikipedia 會比看 conference papers 適宜

看書會比看 journal papers 或 Wikipedia 適宜

(11) 如果實在沒有適合的書籍，可以看 “review”， “survey”，或 “tutorial” 性質的論文

有了相當基礎之後，再閱讀 journal papers

(以 Paper Title， Abstract， 以及其他 Papers 對這篇文章的描述，來判斷這篇 journal papers 應該詳讀或大略了解即可)