

## XV. Applications of Wavelet Transforms

Wavelet 所適用的 applications，通常有以下兩大特點：

- (1) 信號的頻率分佈，會隨著不同的時間(或地點)有較大變異
- (2) Multiscale 的分析扮演重要的角色

Larger sampling interval → ignoring the detail

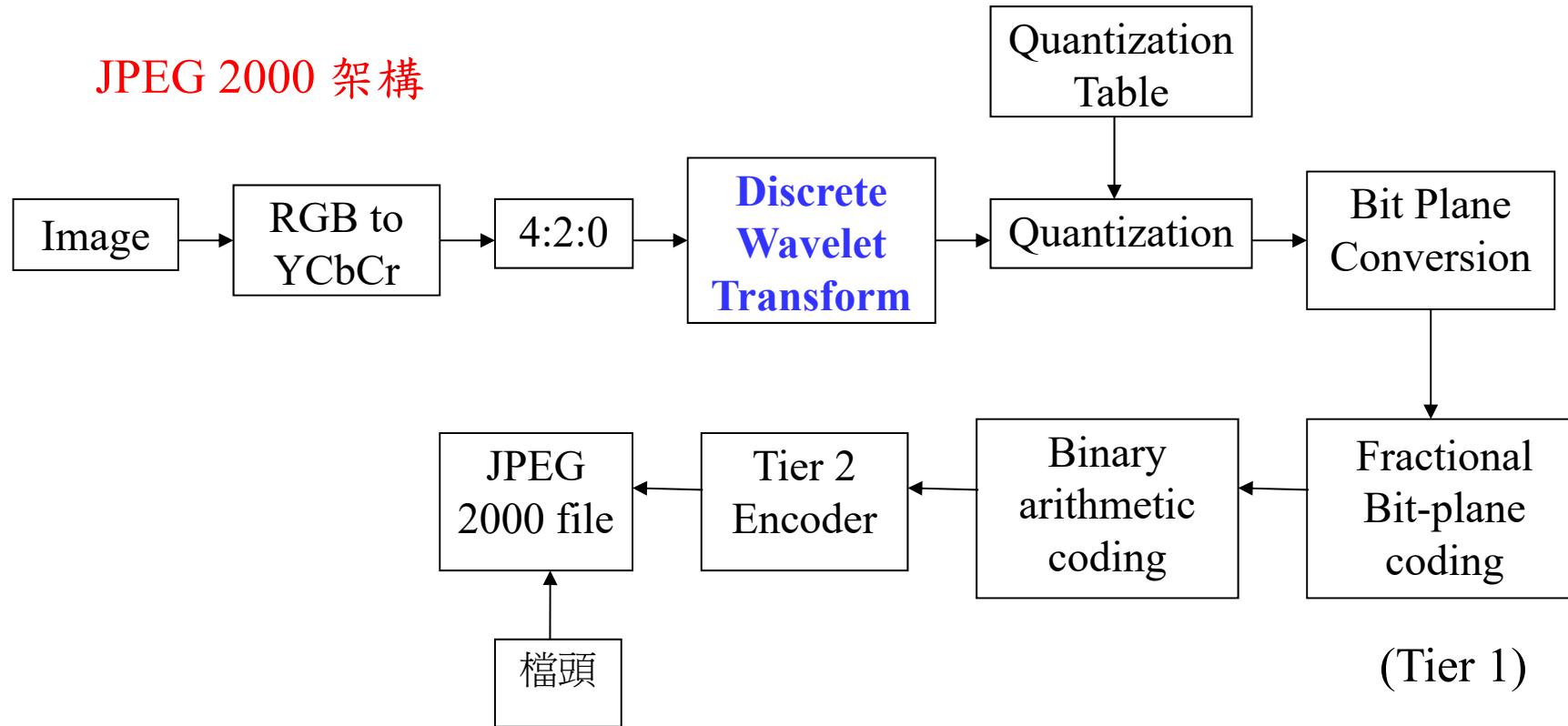
Smaller sampling interval → requiring a lot of data

Wavelet transforms compromise them.

目前，文獻上，80% 以上的應用和 image processing 有關

## (1) Image Compression (JPEG 2000)

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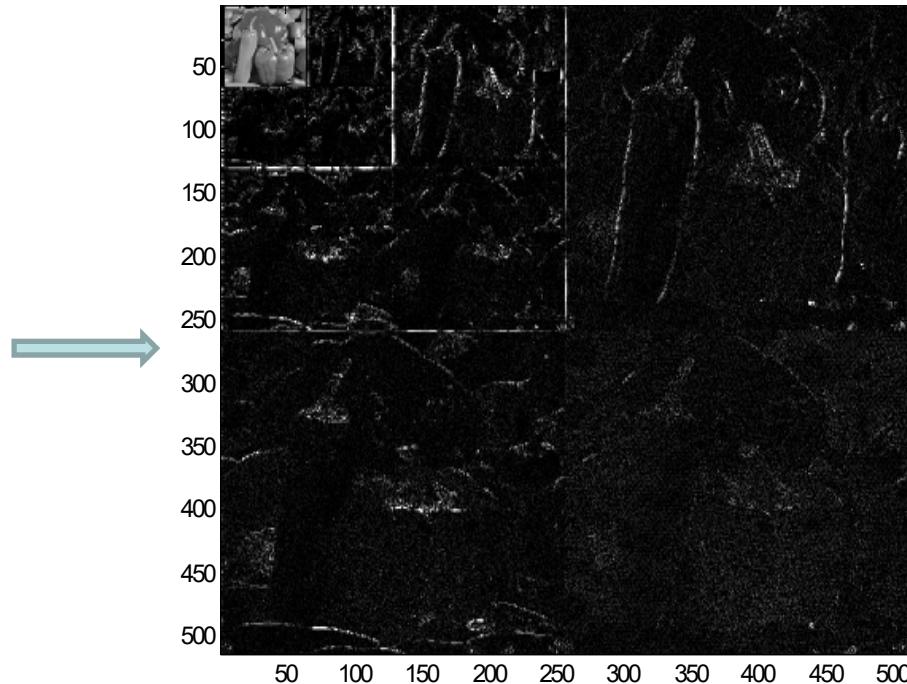
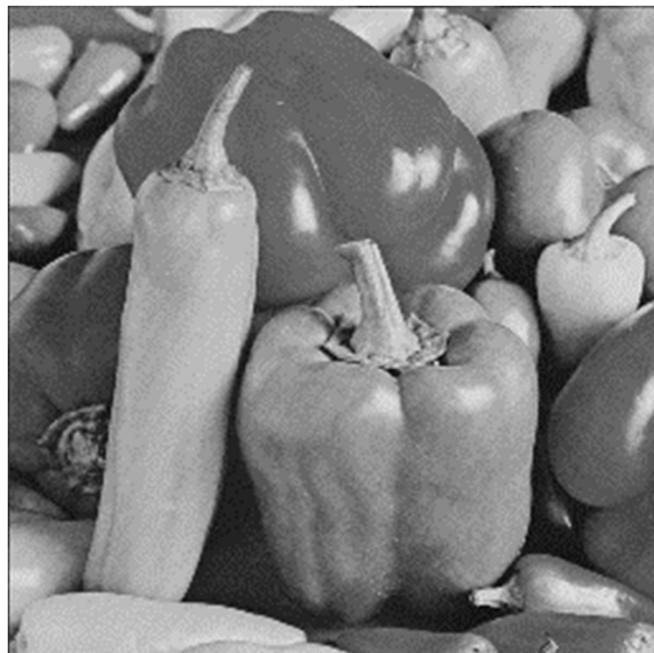
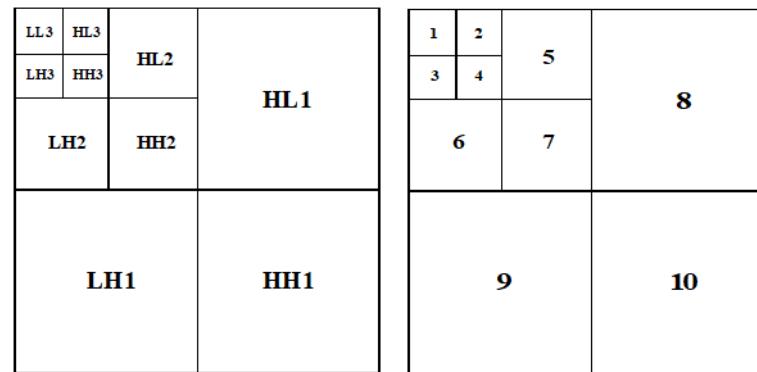


Tier 1: zero coding, sign coding, magnitude refinement coding, run length coding

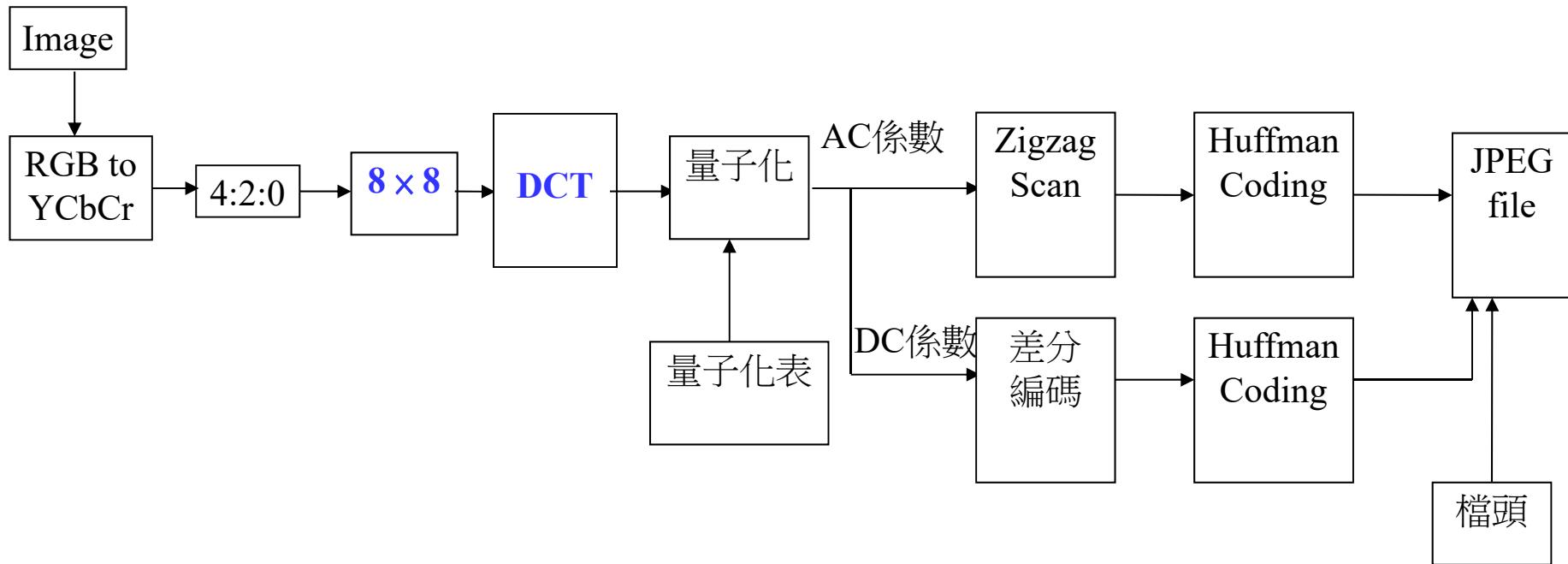
Tier 2: 用以控制檔案大小 (例如只取比較重要的地方編碼)

註：感謝 2010 年修課的潘冠臣同學幫忙整理

## The subbands of the discrete wavelet transform (DWT)



## 比較：傳統 JPEG 架構



問題：由於  $8 \times 8$  的切割，在高壓縮率時會造成 blocking effect

Original image

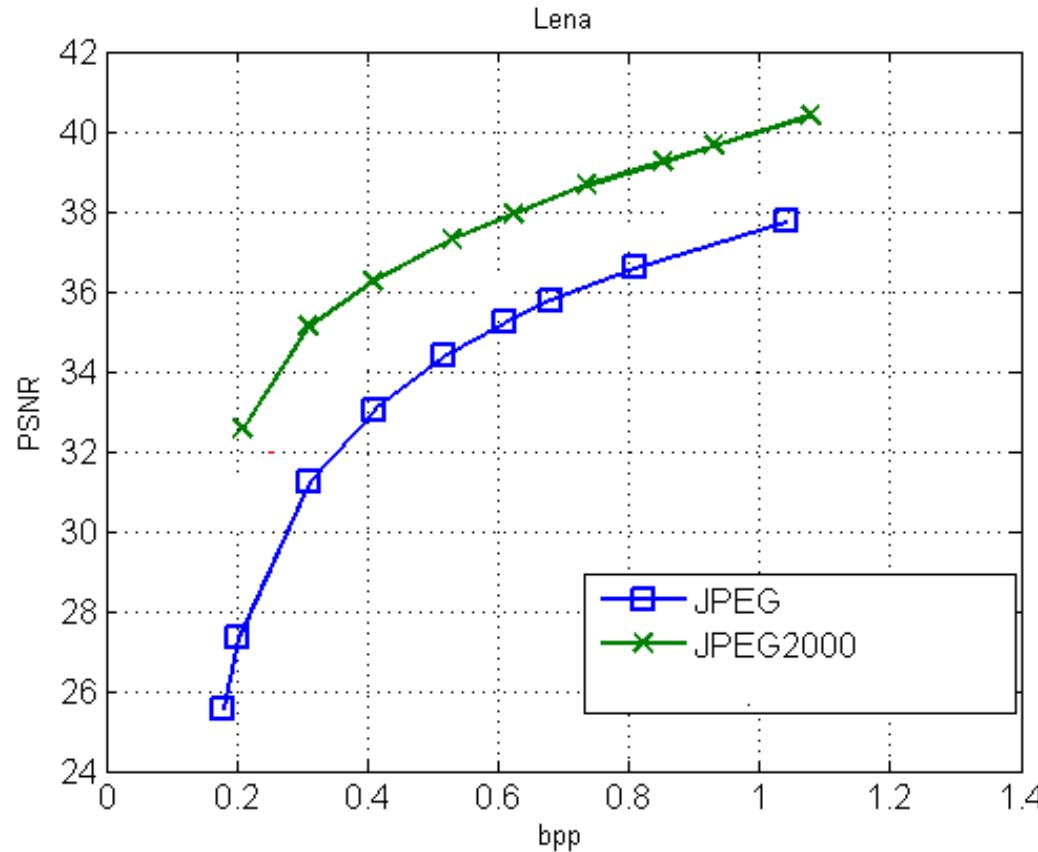
DCT-based  
image compressionWavelet-based  
image compression

CR = 53.4333

CR = 51.3806

CR: compression ratio

註：感謝 2006 年修課的黃俊德同學



bpp: bit per pixel (每一點平均需要多少個 bits)

PSNR: peak signal to noise ratio (PSNR), see page 512

使用 JPEG 2000 做影像壓縮的優點：

- (1)
- (2)
- (3)

所以，在高壓縮率之下，重建的影像仍有不錯的品質

Question:

Why JPEG 2000 has not replaced the status of JPEG now?

參考資料

C. Christopoulos, A. Skodras, and T. Ebrahimi, “The JPEG2000 still image coding system: An overview,” *IEEE Trans. Consumer Electronics*, vol. 46, no. 4, pp.1103-1127, Nov. 2000.

## Another Compression Algorithm: SPIHT

Using the correlation among high frequency parts in different layers

B.J. Kim, Z. Xiong, and W.A. Pearlman. “Low bit-rate scalable video coding with 3-D set partitioning in hierarchical trees (3-D SPIHT),” *IEEE Trans. Circuits Syst. Video Technol.*, vol. 10, pp. 1374-1387, 2000.

## (2) Edge and Corner Detection

## (3) Pattern recognition

(a) Feature extraction

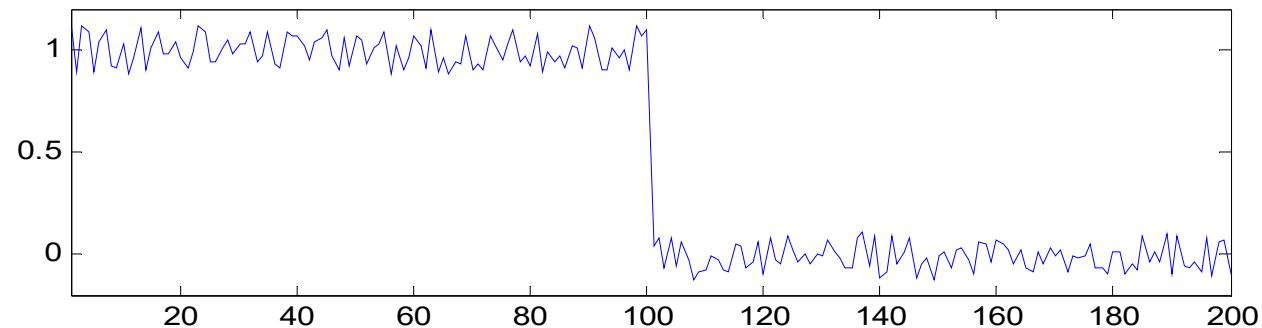
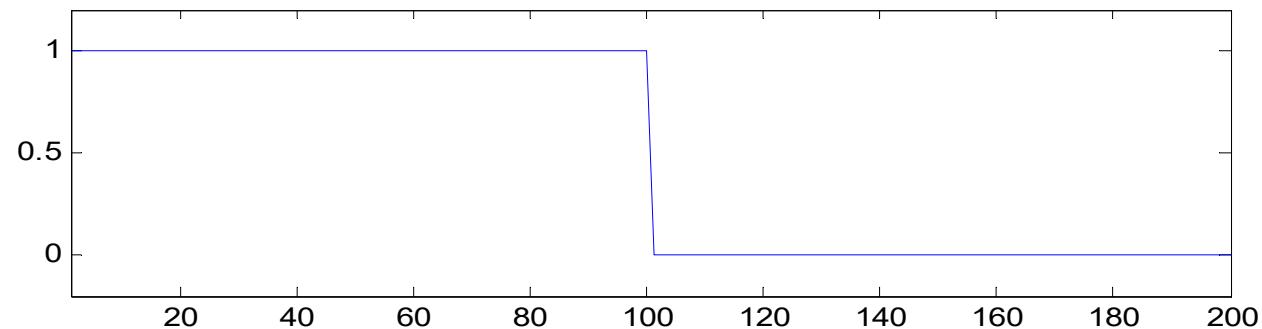
(Using the wavelet features)

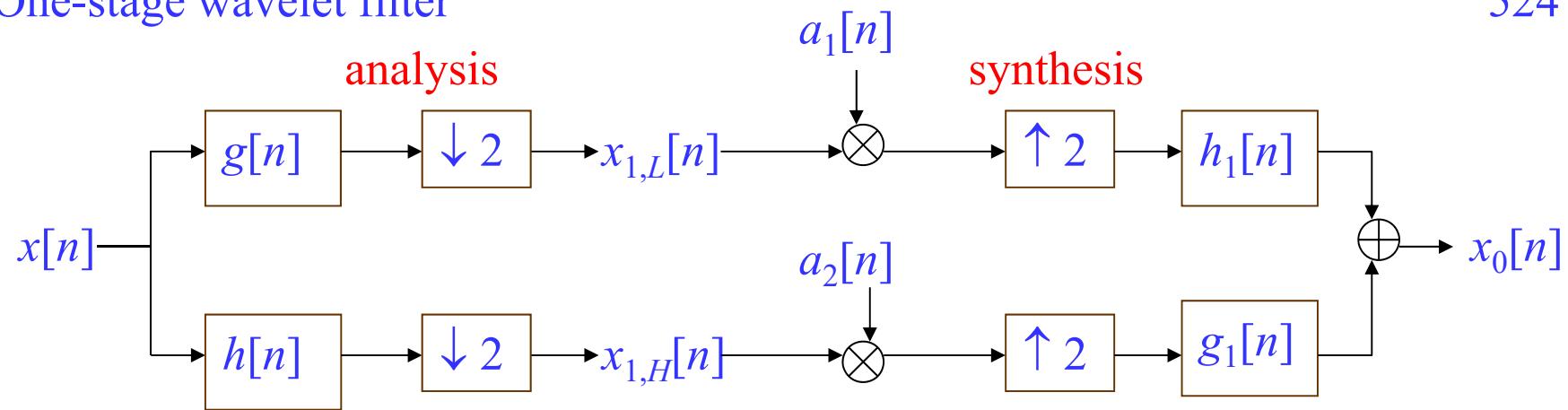
(b) Computation Time 和縮小的 pattern 互相比較 (節省運算)

## (4) 強調前景，壓縮背景

## (5) Filter Design

如何不傷到 edge，又能夠將 noise 去除掉？





(將 page 475 的架構中間加上 transfer functions  $a_1[n]$  and  $a_2[n]$ )

做 filter design 時，可以令

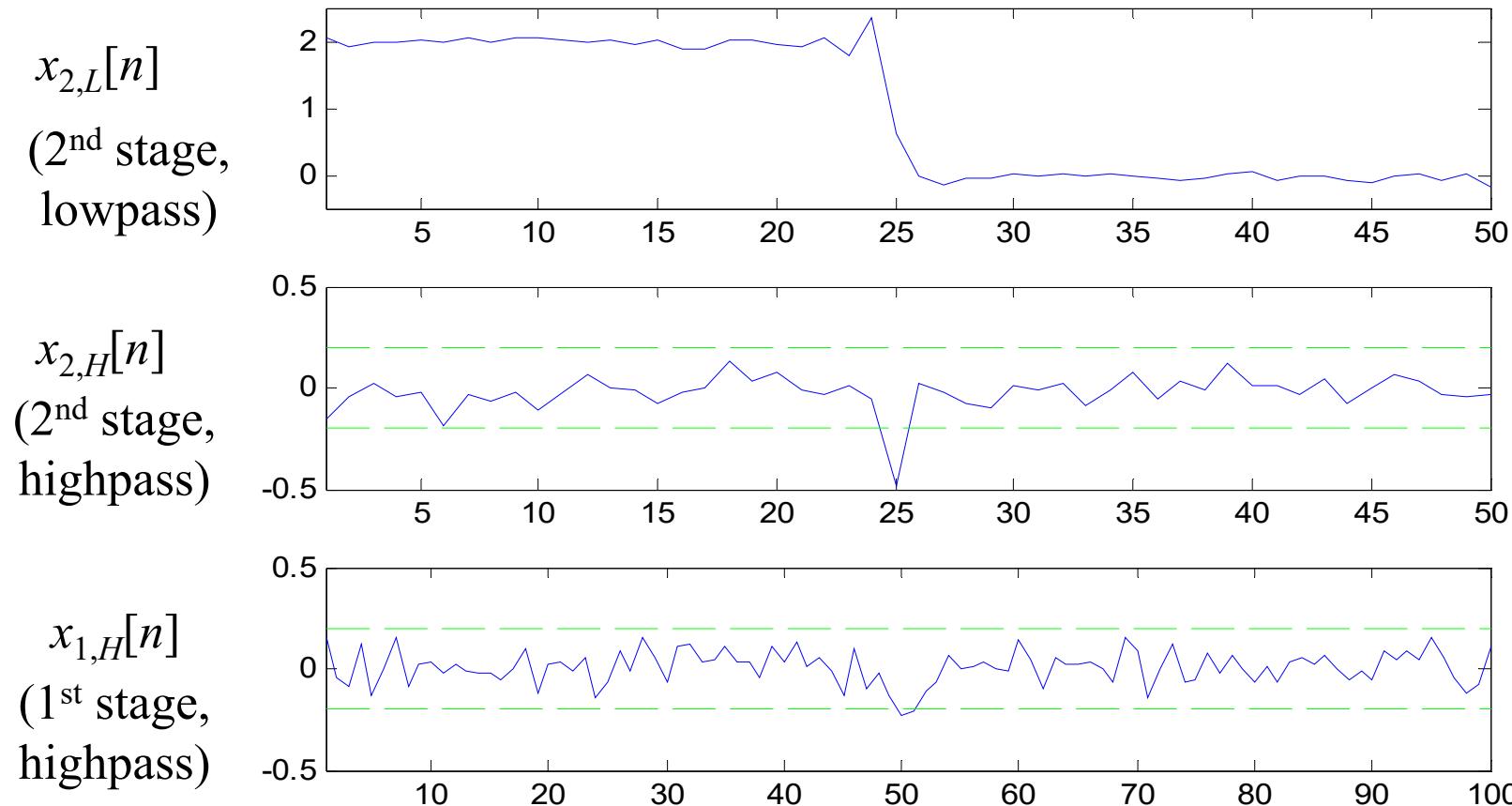
$$a_1[n] = 1,$$

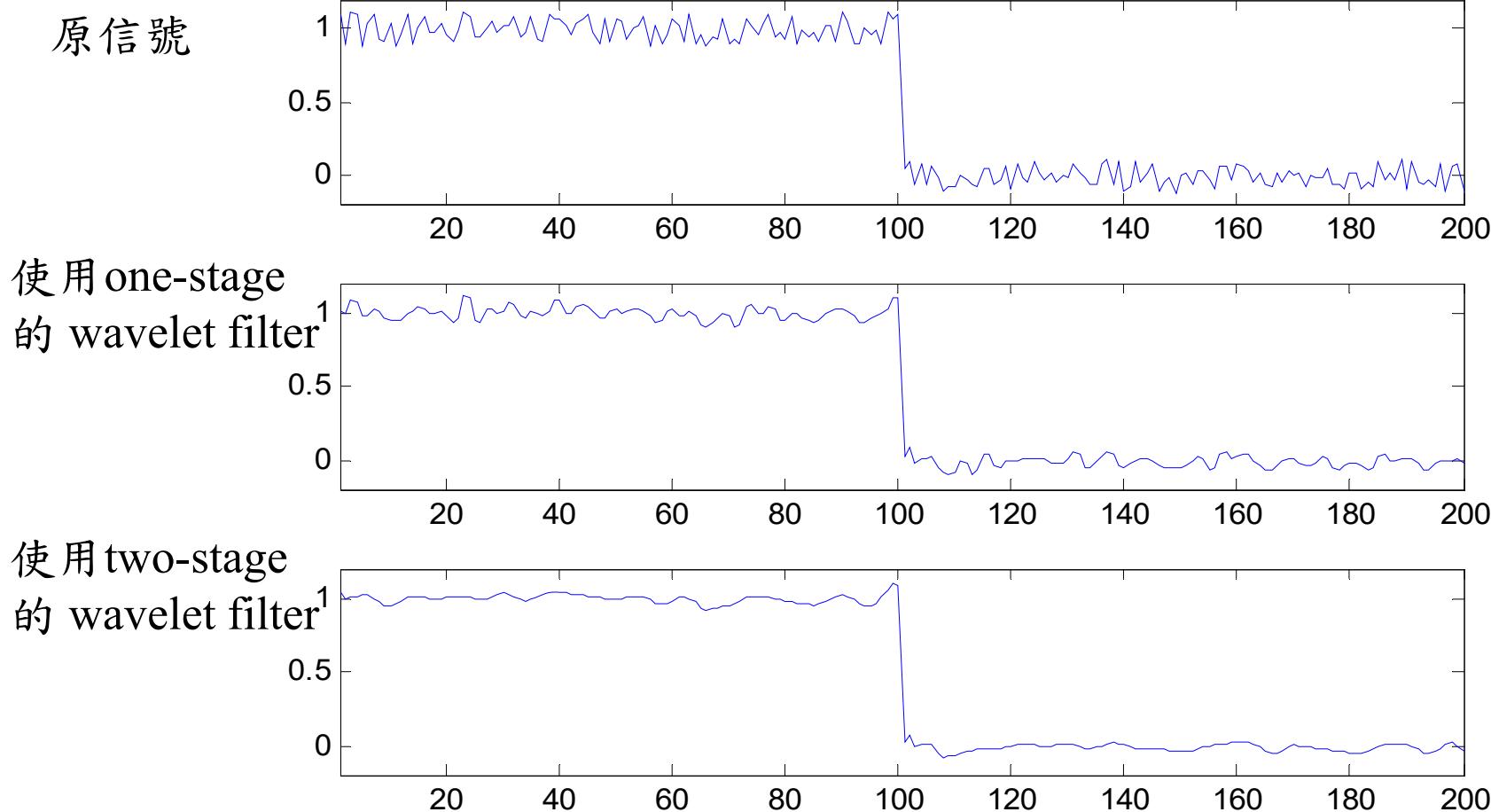
$$a_2[n] = 0 \text{ for non-edge region}$$

$$a_2[n] = 1 \text{ for edge region}$$

以  $x_{1,H}[n]$  的 amplitude  
來區分

必要時可使用 two-stage 以上的 wavelet filter





## (6) Music

音樂當中，音每高一個音階，頻率就增為二倍

音樂 每一音階有12個半音，增加一個半音，頻率增加  $2^{1/12}$  倍

(等比級數)

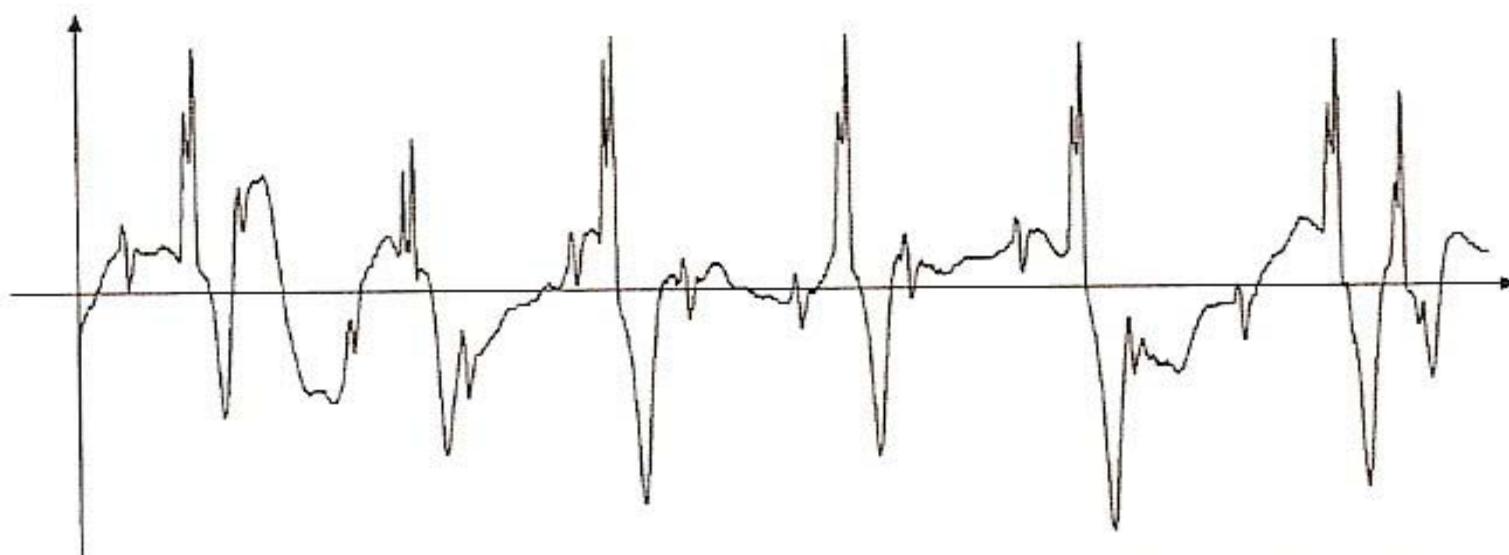
|    | Do  | 升Do | Re  | 升Re | Me  | Fa  | 升Fa | So  | 升So | La  | 升La | Si   |
|----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|------|
| Hz | 270 | 286 | 303 | 321 | 340 | 360 | 382 | 405 | 429 | 454 | 481 | 510  |
| Hz | 540 | 572 | 606 | 642 | 680 | 721 | 764 | 810 | 857 | 908 | 962 | 1019 |

## (7) Acoustics

## (8) Analyzing the Electrocardiogram (ECG)

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- Is the rhythm of the cardiac valve in synchronization with that of the heart muscle?
- Does the heart muscle relax between beats?



From: A. K. Louis, P. Maab, and A. Rieder, “*Wavelets Theory and Applications*”, John Wiley & Sons, Chichester, 1997.

## (9) 「短期因素」和「長期因素」的分析

population

economical data

temperature

## (10) 其他奇奇怪怪的應用

指紋的辨識

羊毛質料的辨識

## Time-frequency Analysis 和 Wavelet 在應用上的異同處

相同：都能夠處理一個信號的頻率分佈會隨時間而改變的情形

不同：Time frequency analysis 對於瞬間頻率的分析比較精確

Wavelet 可作「巨觀」和「微觀」的分析

由於 memory requirement 較少，適合 2D 的  
image analysis 和 3D 的 video analysis

## 附錄十五 希臘字母大小寫與發音一覽表

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|       |       |       |       |        |          |       |      |       |
|-------|-------|-------|-------|--------|----------|-------|------|-------|
| 大寫    | A     | B     | Γ     | Δ      | E        | Z     | H    | Θ     |
| 小寫    | α     | β     | γ     | δ      | ε        | ζ     | η    | θ     |
| 英文拚法  | alpha | beta  | gamma | delta  | epsilon  | zeta  | eta  | theta |
| KK 音標 | ˋælfə | ˋbetə | ˋgæmə | ˋdeltə | ˋepsələn | ˋzetə | ˋitə | ˋθitə |

|       |        |       |        |     |    |     |          |     |
|-------|--------|-------|--------|-----|----|-----|----------|-----|
| 大寫    | I      | K     | Λ      | M   | N  | Ξ   | O        | Π   |
| 小寫    | ι      | κ     | λ      | μ   | ν  | ξ   | ο        | π   |
| 英文拚法  | iota   | kappa | lambda | mu  | nu | xi  | omicron  | pi  |
| KK 音標 | ai`otə | ˋkæpə | ˋlæmdə | mju | nu | ksi | ˋamikran | pai |

| 大寫    | P      | $\Sigma$ | T      | Y          | $\Phi$          | X      | $\Psi$ | $\Omega$         |
|-------|--------|----------|--------|------------|-----------------|--------|--------|------------------|
| 小寫    | $\rho$ | $\sigma$ | $\tau$ | $\upsilon$ | $\varphi, \phi$ | $\chi$ | $\psi$ | $\varpi, \omega$ |
| 英文揅法  | rho    | sigma    | tau    | upsilon    | phi             | chi    | psi    | omega            |
| KK 音標 | ro     | `sigmə   | taʊ    | ˈjupsələn  | fai             | kai    | sai    | ˈomɪgə           |

## 1. Directional Form 2-D Wavelet Transforms

一般的 2-D wavelet transform，其實可分解成沿著  $x$ -axis 以及沿著  $y$ -axis 的 1-D wavelet transforms 的組合

其實，2-D wavelet transform 不一定要沿著  $x$ -axis， $y$ -axis 來做

Directional 2-D wavelet transforms:

- curvelet
- contourlet
- bandlet
- shearlet
- Fresnelet
- wedgelet
- brushlet

- **Curvelet (ridgelet)**

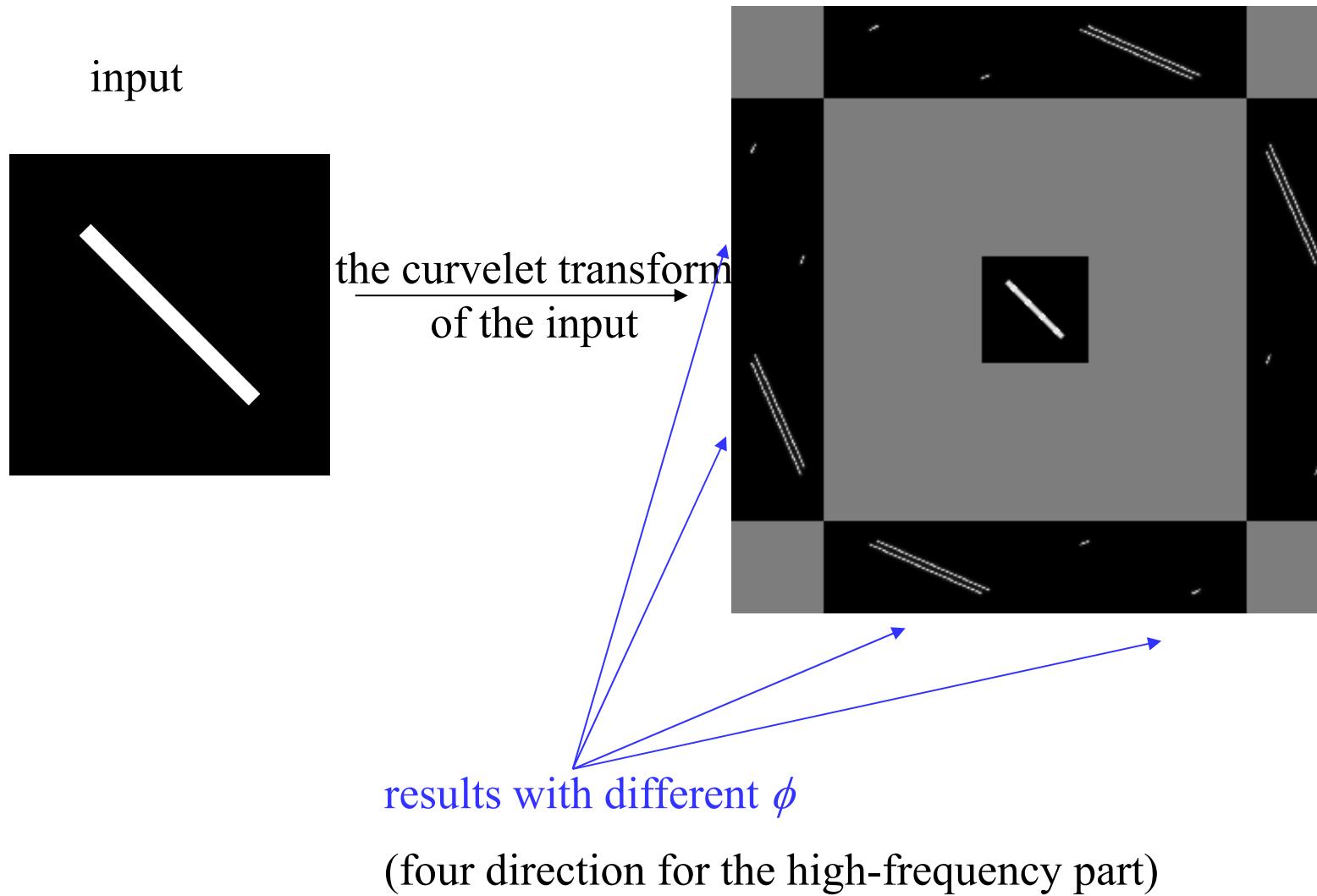
$$F_w(a, b, \phi) = \frac{1}{\sqrt{b}} \int_{-\infty}^{\infty} f(r \cos \phi, r \sin \phi) \psi\left(\frac{r-a}{b}\right) dr$$

rotation

比較：原本的 1-D wavelet

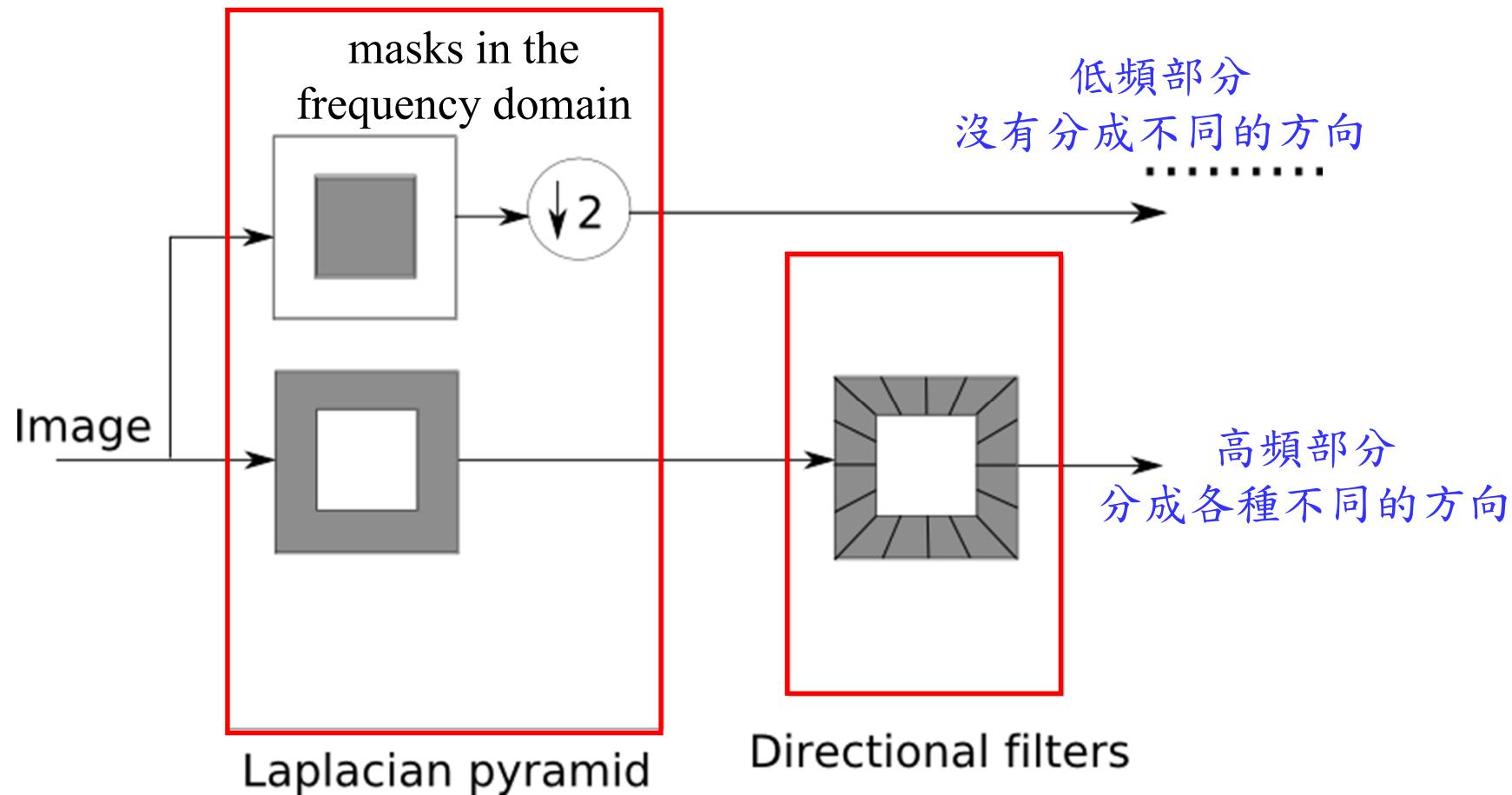
$$F_w(a, b) = \frac{1}{\sqrt{b}} \int_{-\infty}^{\infty} f(x) \psi\left(\frac{x-a}{b}\right) dx$$

E. Candès and D. Donoho, "Curvelets – a surprisingly effective nonadaptive representation for objects with edges." In: A. Cohen, C. Rabut and L. Schumaker, Editors, *Curves and Surface Fitting*: Saint-Malo 1999, Vanderbilt University Press, Nashville (2000), pp. 105–120.



## • Contourlet

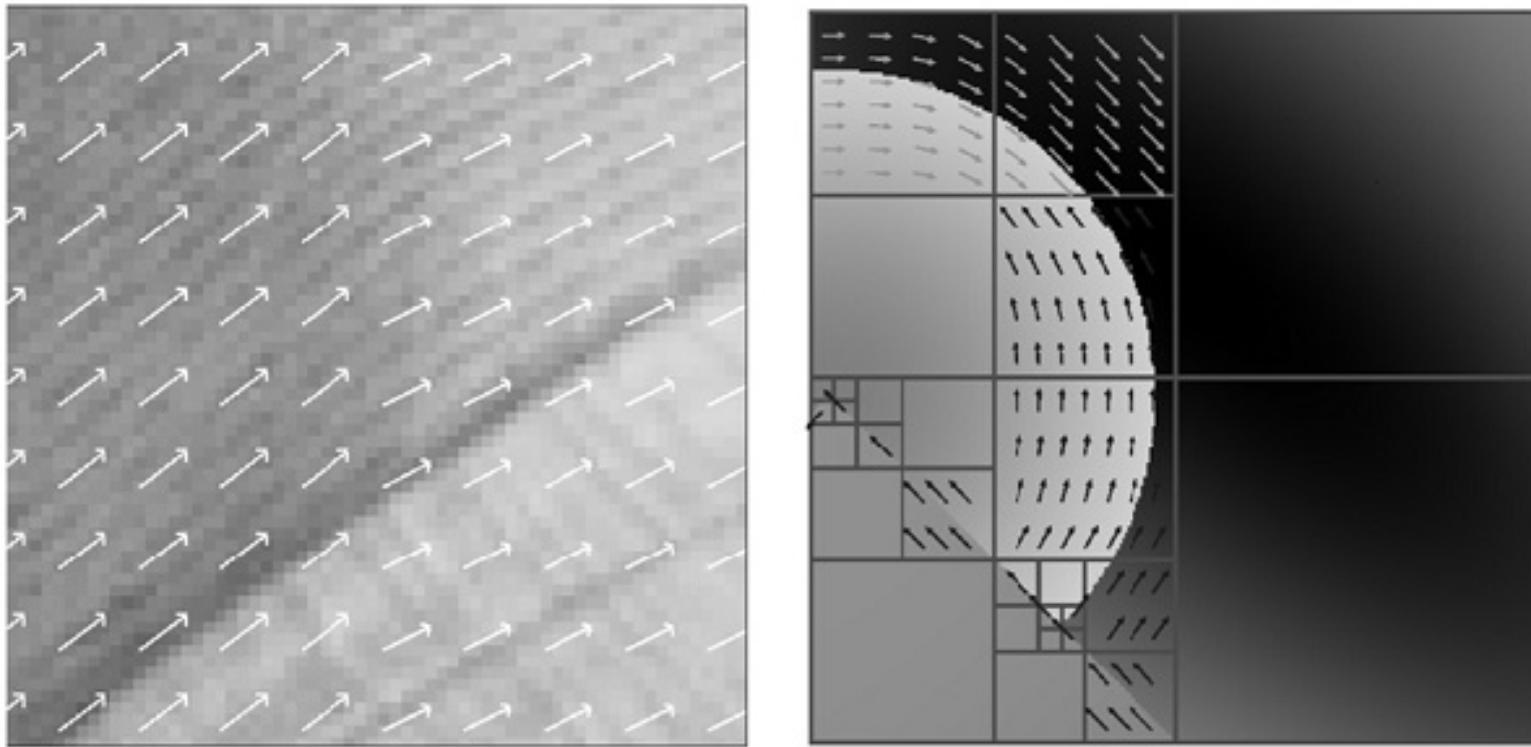
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M. Do and M. Vetterli, "The contourlet transform: An efficient directional multiresolution image representation," *IEEE Trans. Image Processing*, vol.14, no.12, pp.2091–2106, Dec. 2005.

- Bandlet

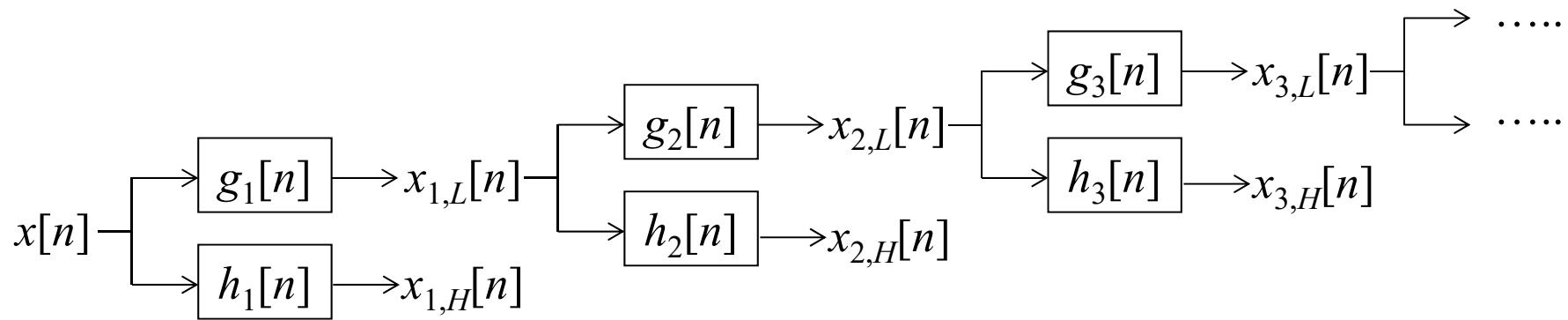
根據物體的紋理或邊界，來調整 wavelet transforms 的方向



Stephane Mallet and Gabriel Peyre, "A review of bandlet methods for geometrical image representation," *Numerical Algorithms*, Apr. 2002.

## 2. Stationary Wavelet Transforms

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其中  $g_j[n] \rightarrow \uparrow 2 \rightarrow g_{j+1}[n]$        $h_j[n] \rightarrow \uparrow 2 \rightarrow h_{j+1}[n]$

Q: 和原本 discrete wavelet transform 不一樣的地方在哪裡？

G. P. Nason and B. W. Silverman, “The stationary wavelet transform and some statistical applications,” *Lecture Notes in Statistics*, available in <http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.49.2662&rep=rep1&type=pdf>

### 3. Bandwidth Form Wavelet Transforms

A little modification for  $g[n]$  and  $h[n]$

### 4. Multi-Band Wavelet Transforms

Instead of only two outputs

# Happy New Year!

祝各位期末考順利，寒假愉快！