Homework 5 (Due: 6/19th)

(1) Write the Matlab or Python code to compute the FFT of two N-point real signals x and y using only one N-point FFT. (20 scores)

$$[Fx, Fy] = \text{fftreal}(x, y)$$

The code should be handed out by NTUCool.

(2) Suppose that length($x[n]$) = 1200. What is the best way to implement the convolution of $x[n]$ and $y[n]$ if

(a) length($y[n]$) = 300, (b) length($y[n]$) = 30,

(c) length($y[n]$) = 8, and (d) length($y[n]$) = 2?

Please show (i) the calculation method (direct, non-sectioned convolution, or sectioned convolution), (ii) the number of points of the FFT, (iii) and the number of real multiplications for the best implementation method. Also, consider the general case where $x[n]$ and $y[n]$ are complex sequences and the FFT of $y[n]$ can be computed in prior. (25 scores)

(3) (a) What are the number of entries equal to 1 and -1 for the 2^k-point Walsh transform? (b) What are the number of entries equal to 1, 0, and -1 for the 2^k-point Haar transform? (c) What is the most important application of the Walsh transform nowadays? (d) What is the most important advantage of the Haar transform nowadays? (20 scores)

(4) (a) What is the results of CDMA if there are three data [1 0 1], [1 1 0], [0 1 1] and these three data are modulated by the 1st, 4th, and 10th rows of the 16-point Walsh transform? (The beginning row is the 1st row). (10 scores)

(b) In (a), if the 7th and the 19th entries of the CDMA results are missed, can we recover the original data? Why? (5 scores)

(5) Ramanujan's Sum in NTT

Given $M = 11$, $\alpha = 8+6i$, and $N = 12$. Please determine the complex number theoretic transform (CNT) of **x** if

$$\mathbf{x} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Hint: fft(x) is as follows, which is Ramanujan's Sum

$$fft(\mathbf{x}) = \begin{bmatrix} 4 & 0 & 2 & 0 & -2 & 0 & -4 & 0 & -2 & 0 & 2 & 0 \end{bmatrix}$$ (8 scores)

(6) (a) Please determine

$$3^{2049} \bmod 103 \qquad \text{(Hint: 費馬小定理)}$$

(b) Suppose that $x \bmod 43 = 2$ *and* $x \bmod 67 = 13$

Please Determine

$$x \bmod 2881. \qquad \text{(Hint: Chinese Remainder Theorem)}$$

(c) $n! = n(n-1)(n-2) \ldots 1$. Please determine 39! mod 43

(Hint: Wilson's Theorem) (12 scores)

(Extra): Answer the questions according to your student ID number.

(ended with (1, 6), (2, 7), (3, 8), (4, 9))