

Homework 1 (Due: March 24th)

(1) Find the solutions of the following nonlinear DEs. (30 scores)

(a) $y''(x) + x(y'(x))^2 = 0, \quad y(1) = -2, \quad y'(1) = 2$

(b) $y''(x) + (y'(x))^2 + y'(x)y(x) = 0, \quad y(0) = 0 \quad y'(0) = 1$

(c) $y''(x) + \exp(-2y(x)) = 0, \quad y(0) = 0 \quad y'(0) = 1$

(2) Solve the following PDEs. (40 scores)

(a) $x \frac{\partial u(x, y)}{\partial x} + y \frac{\partial u(x, y)}{\partial y} = u(x, y)$

(b) $\frac{\partial^2 u(x, t)}{\partial x^2} = \frac{\partial u(x, t)}{\partial t} \quad 0 < x < 2, \quad t > 0,$

$u(0, t) = u(2, t) = 0 \quad \text{for } t > 0, \quad u(x, 0) = 1 - (x - 1)^2 \quad \text{for } 0 < x < 2$

$$(c) \frac{\partial^2 u(x, y)}{\partial x^2} + \frac{\partial^2 u(x, y)}{\partial y^2} = 0 \quad 0 < x < 1, \quad 0 < y < 1$$

$$\left. \frac{\partial}{\partial x} u(x, y) \right|_{x=0} = \left. \frac{\partial}{\partial x} u(x, y) \right|_{x=1} = 0 \quad \text{for } 0 < y < 1,$$

$$\left. \frac{\partial}{\partial y} u(x, y) \right|_{y=0} = 0 \quad \text{for } 0 < x < 1, \quad u(x, 1) = \sin^2(3\pi x) \quad \text{for } 0 < x < 1,$$

$$(d) \frac{\partial u(x, y)}{\partial x} + 2 \frac{\partial u(x, y)}{\partial y} = x + \cos y$$

(3) Solve the following 1st order nonlinear DE numerically. The codes should be handed out by NTUCool. (30 scores)

$$\frac{\partial y(x)}{\partial x} = \cos(xy(x)) \exp(-x^{1/3}), \quad y(0) = 1, \quad 0 \leq x \leq 10, \quad x_{n+1} - x_n = 0.02$$

(a) By Euler's method.

(b) By modified Euler's method.

(c) By the RK4 method.