Homework 2 (Due: April 14th)

(1) Solve the following PDEs. (30 scores)

(a) $$x^2 \frac{\partial u(x,y,z)}{\partial x} + y \frac{\partial u(x,y,z)}{\partial y} + \frac{\partial u(x,y,z)}{\partial z} = 0$$

(b) $$3\frac{\partial u(x,y,z)}{\partial x} + 2\frac{\partial u(x,y,z)}{\partial y} + \frac{\partial u(x,y,z)}{\partial z} = x + y + z$$

(c) $$\frac{\partial u^2(x,y,t)}{\partial x^2} + \frac{\partial u^2(x,y,t)}{\partial y^2} = \frac{\partial u^2(x,y,t)}{\partial t^2}$$

$$u(0,y,t) = 0, \quad u(2,y,t) = 0, \quad 0 < y < 2, \quad t > 0$$

$$u(x,0,t) = 0, \quad u(x,2,t) = 0, \quad 0 < x < 2, \quad t > 0$$

$$u(x,y,0) = \left(1 - |x-1|\right)\left(1 - |y-1|\right), \quad 0 < x < 2, \quad 0 < y < 2.$$

$$\left.\frac{\partial}{\partial t} u(x,y,t)\right|_{t=0} = 0, \quad 0 < x < 2, \quad 0 < y < 2.$$

(2) (a) Find the steady-state temperature $u(r, \theta)$ in the fan plate where

$$u(2,\theta) = 2, \quad -\pi/6 < \theta < \pi/3$$

$$u(r,-\pi/6) = u(r,\pi/3) = 1, \quad 0 < r < 2.$$

(Hint): Set $v(r,\theta) = u(r,\theta - \pi/6) - 1$ (20 scores)

(b) Find the general solution of the steady-state temperature $u(r, z)$ in the cylinder where

$$u(r,0) = u(r,1) = 0, \quad 0 \le r < 2, \quad u \text{ is independent of } \theta.$$

(3) Solve the following PDE by the 1-sided Laplace transform.

(10 scores)

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \quad 0 < x < 10, \quad t > 0$$

$$u(0,t) = 0, \quad u(10,t) = 0, \quad t > 0$$

$$u(x,0) = \sin(\pi x / 10), \quad 0 < x < 10.$$

(4) (a) Convert 1, x, and x^2 into an orthonormal function set for $x \in [0, \pi]$.

(b) Expand $g(x) = \exp(x/\pi)$ for $x \in [0, \pi]$ by $q(x) = c_0 + c_1 x$

such that $\int_0^{\pi} \left(g(x) - q(x) \right)^2 dx$ is minimized. (20 scores)

(5) Suppose that there is a set of five 'discrete' basis.

$$b_k[n] = \cos(k|n|^{1.25}) \qquad n = 0, 1, 2, \cdots, 12$$
$$k = 0, 1, 2, 3$$

(a) Use the Gram-Schmidt method (written by Matlab or Python) to convert $b_k[n]$ (k = 0~3) into an orthonormal basis set. Please plot the four derived orthonormal basis.

(b) Use the Gram-Schmidt method (written by Matlab or Python) to convert $b_k[n]$ ($k = 0 \sim 3$) into an orthonormal basis set if the weight is

$$w[n] = \exp(-n/5)$$

Please plot the four derived orthonormal basis.

The codes should be handed out by NTUCool. (20 scores)