

### Homework 3 (Due: May 5<sup>th</sup>)

(1) Determine the Fourier transform of the following functions.

(a)  $g(x) = \cos(10\pi x)$  for  $0 < x < 20$ ,  $g(x) = 0$  otherwise.

(b)  $g(x) = x \exp(-(x-1)^2)$ . (20 scores)

(2) Determine the following convolutions.

(a)  $\delta'(x) * \sin(\pi x) * \text{sinc}(3x) * \text{sinc}(4x) * \text{sinc}(5x)$

(b)  $x \exp(-\pi x^2) * \sin(4\pi x)$

(c)  $\exp(-|x|^{0.5}) * \delta(\sin(\pi x))$  (30 scores)

(3) Determine the 2D Fourier transform of

$g(x, y) = 1$  for  $(x+1)^2 + y^2 < 4$ ,  $g(x, y) = 0$  otherwise. (10 scores)

(20 points)

- (4) With a computer program or not,  
(a) find the determinant of this 5 by 5 matrix modulo 7,  
meaning that the answer is in  $\{0, 1, 2, 3, 4, 5, 6\}$ .

$$\mathbf{C} = \begin{bmatrix} 5 & 1 & 1 & 2 & 1 \\ 5 & 1 & 1 & 1 & 2 \\ 1 & 5 & 1 & 1 & 2 \\ 1 & 5 & 1 & 1 & 1 \\ 2 & 1 & 5 & 1 & 1 \end{bmatrix}$$

- (b) Then, find the determinants of  $\mathbf{C} - 1\mathbf{I}$ ,  $\mathbf{C} - 2\mathbf{I}$ ,  $\mathbf{C} - 3\mathbf{I}$ ,  $\mathbf{C} - 4\mathbf{I}$ ,  $\mathbf{C} - 5\mathbf{I}$ .  
Note that  $1 - 2 = 6$  in the modulo-7 world.

Your answer should coincide with  $f(0), f(1), f(2), f(3), f(4)$ ,  
where  $f$  is the characteristic polynomial of  $\mathbf{C}$  modulo 7.

With this in mind, enumerate all monic quintic polynomials  
whose coefficients are 0~6 and evaluate them at 0~4 to find  $f$ .

(5) Using a Matlab or Python code to determine the continuous Fourier transform of the following functions by the DFT.

(a)  $g(x) = \exp(-|x|^{0.5})$  for  $-2 < x < 2$ ,  $g(x) = 0$  otherwise,  $\Delta_x = 0.05$ .

(b)  $g(x) = \cos(2x^2)$  for  $-3 < x < 3$ ,  $g(x) = 0$  otherwise,  $\Delta_x = 0.02$ .

The codes should be handed out by NTUCool. (20 scores)