Homework 4 (Due: May 26th)

(30 points)

(1) The problem of compressed sensing with sparsity k is to solve

$$Ax \approx y$$

by minimizing

$$|Ax - y|^2$$

among all vectors x such that $|x|_0$ is at most k.
The zero-norm is the number of nonzero elements, called the *sparsity* of x.
The two-norm is sometimes called the *residual* norm.

Now consider this matrix and this vector

$$A = \begin{bmatrix} 365 & 60 & 24 \\ 365 & -60 & 0 \\ 0 & 0 & 7 \end{bmatrix} \qquad y = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

(a) Which x gives the best approximation for sparsity 1? What is the residual?
(b) Which x gives the best approximation for sparsity 2? What is the residual?
(c) Which x gives the best approximation for sparsity 3? What is the residual?

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(2) With a computer program or not,

(a) find the Jordan decomposition of the following matrix

$$\mathbf{\Gamma} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

Its eigenvalues are usually denoted as Φ and φ, where $\Phi > \varphi$.

(b) Inspect $\Gamma, \Gamma^2, \Gamma^3, \ldots$ and conclude that Γ^n consists of four Fibonacci numbers $F_?$, $F_?$, $F_?$ and $F_?$. Use 6.3.2 to infer that $F_n \sim \text{const} \cdot \Phi^n$

What do you get if you use 6.3.2 to evaluate F_{100}?
What do you get if you simply use arbitrary-length integer and the classical recursive formula $F_{n+1} = F_n + F_{n-1}$ to evaluate F_{100}?

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(3) Program problem. Remember to upload both result and source code.

```
............
AACCAAACCAA
..CC...CC..
AACCAAACCAA
..CC...CC..
AACCAAACCAA
..CC...CC..
..CCCCCCC..
..GGGGGGG..
```

This is a finned heatsink (散熱塔).
Each character represents a square that exchange heat with its 4 neighbors (not 8).
Dot is air. Heat conductance 0.01°C/°C-edge-ms
A is Al. Heat conductance 0.1°C/°C-edge-ms
C is Cu. Heat conductance 0.2°C/°C-edge-ms
G is GPU. Heat conductance 0.05°C/°C-edge-ms
The heat conductance between two different materials is the harmonic average of the two conductance.

For example, if A is next to another A, one 30°C and the other 50°C, then they become 32°C and 48°C the after one ms.
(Because the heat difference is 20°C, they exchange 20 * 0.1 = 2°C.)
Everything begin with 25°C. GPU generates heat at 1°C/ms.
Air is fixed at 25°C. No heat exchange with outside world.

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Use $x_{t+1} = Ax_t + b$ to describe the evolution of heat.

(a) Write a 100 word paragraph to describe how you obtain A.

(b) Use Euler method with time step 1ms to plot something like the one below. What is the stable temperature of the hottest part of GPU (precision requirement is 0.1 °C).

(c) Solve the linear equation $x = Ax + b$ to get the stable temperature (precision 0.01 °C)

(d) Find the top ten eigenvalues of A. (If your A is not symmetric, something is wrong.)

(e) Let s be the largest eigenvalue of A (or the second largest if the largest is 1). Compute $m = \log_s(0.01)$ to conclude that it takes m ms to reduce the temperature difference to 1%.