Homework 4 (Due: May 28th)

(1) Determine the circular convolution of $x[n]$ and $h[n]$ where   (10 scores)

$$x[n] = [1, 0, 2, 3, -1, 2, 2, 1, 0], \qquad h[n] = [2, 2, 1, 1, 0, 0, 0, 0, 0]$$

and $N = 9$.

(2) Suppose that the PDF of X is   (20 scores)

$$f_X(x) = x/2 \quad for \ 0 < x < 2, \qquad f_X(x) = 0 \quad otherwise.$$

(a) Determine the CDF and the mean of X.

(b) Determine the standard deviation, the variance, and the skewness of X.

(3) Suppose that the mean, the variance, the skewness, and the kurtosis of X are μ, v, s, and k, respectively. Also suppose that $Y = X/5 + 5$.

(a) Determine the correlation between X and Y.

(b) Determine the mean, the variance, the skewness, and the kurtosis of Y.

(20 scores)

(4) Suppose that the joint PDF of X and Y is: (10 scores)

$$f_{X,Y}(x,y) = \begin{cases} \dfrac{1}{100}\delta(y+2x-1) & for \quad 0 < x < 100 \\ \\ 0 & otherwise \end{cases}$$

Determine the correlation of X and Y.

(5) We assume that   (10 scores)

$$\mathbf{A} = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{bmatrix}$$

Find the permutation matrices $\mathbf{J_1}$ and $\mathbf{J_2}$ such that

$$\mathbf{A} \otimes \mathbf{B} = \mathbf{J_1}(\mathbf{B} \otimes \mathbf{A})\mathbf{J_2}$$

Hints: Expand the Kronecker products and match their entries.

(6) For a real number α and an integer $N \geq 2$, we define the vector

$$\mathbf{v}(\alpha) = \begin{bmatrix} 1 & \alpha & \alpha^2 & \cdots & \alpha^{N-1} \end{bmatrix}^T$$

Find the value of the following norms:

$$\|\mathbf{v}(\alpha)\|_0, \quad \|\mathbf{v}(\alpha)\|_1, \quad \|\mathbf{v}(\alpha)\|_2, \quad \|\mathbf{v}(\alpha)\|_\infty, \quad \|\mathbf{v}(\alpha)\mathbf{v}^T(\alpha)\|_F$$

Simplify and express your answers in terms of N and α. Do not use dots
(...) or summation (Σ) in your expression.   (10 scores)

(7) Suppose that   (20 scores)

$$P_{X,Y}(m,n) = \frac{100 - |m-n|}{666700} \qquad for \ m=1,2,\cdots,100, \ n=1,2,\cdots,100$$

Use a Matlab or Python code to determine the cross entropy of X
and Y if

(a) The definition of the cross entropy on page 532 is applied.

(b) The definition of the cross entropy on page 534 is applied.

The code should be handed out by NTUCool.