Homework 5 (Due: June 16th)

(1) Suppose that the length of x is $2n$. Let P be the matrix that puts even rows to the front and the odd rows to the back. How to use numpy or Matlab syntax to get the effect of matrix-vector multiplication Px without actually constructing P? (In particular, your code should cost linear time, not quadratic time.) (10 points)

(2) Suppose that the length of y is n. Let D be the diagonal matrix with the geometric sequence $1, \omega, \omega^2, \ldots$ on the diagonal. How to use numpy or Matlab syntax to get the effect of Dy without actually constructing D?

(10 points)

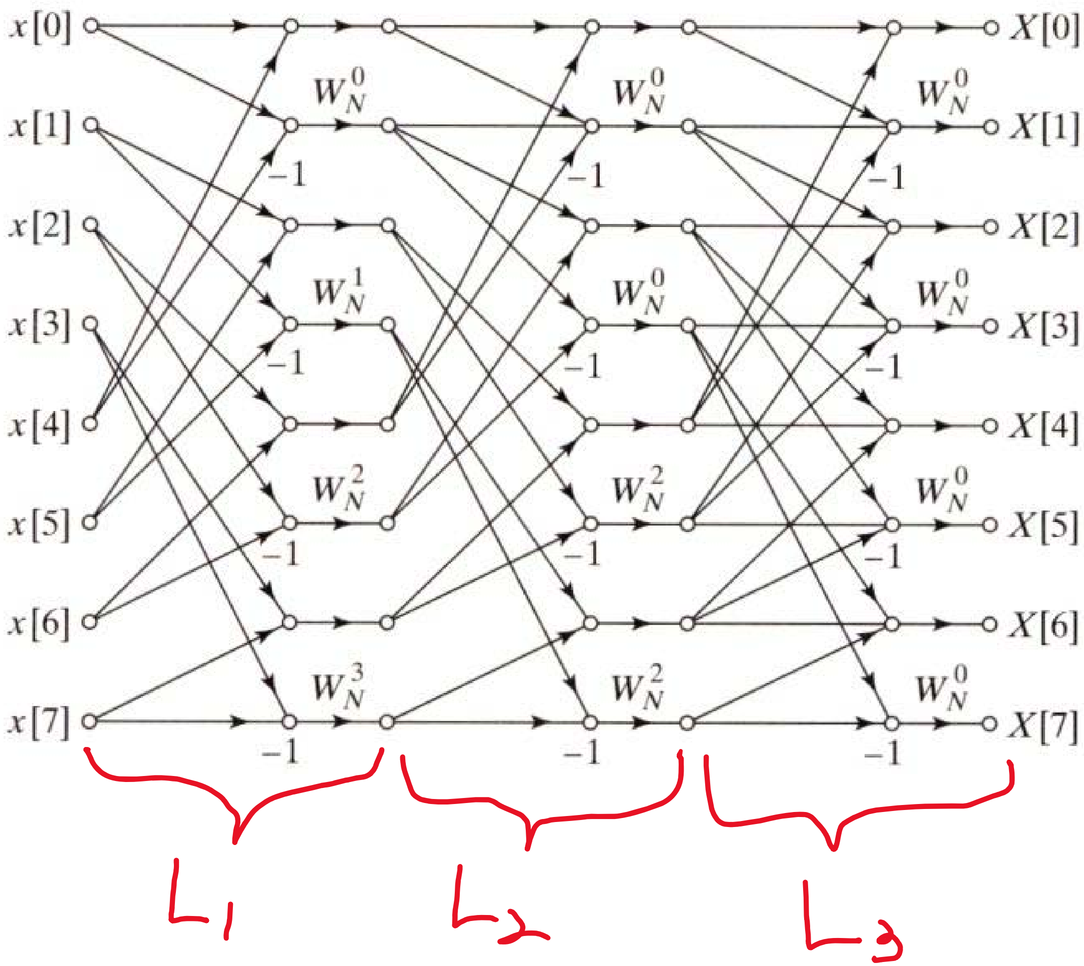
(3) Suppose that the length of z is $2n$. How to use numpy or Matlab syntax to get the effect of (10 points)

$$\begin{bmatrix} I & D \\ I & -D \end{bmatrix} z$$

without actually constructing this matrix?

PS: For each problem on this page, the expected answer is 1~10 lines of codes plus 50 words of explanation. Explanation is very important because it earns partial credit should your code be incorrect. Also, please do not upload the code files. Instead, directly paste your codes on your word file.

(4) According to Wikipedia, the network to the right computes the FT of x. Branching out means data copy. Merging means addition. $W_N^?$ means powers of the eighth root of unity.



Write down three 8x8 matrices that correspond to the effects of the first, second, and third layers of the network. PS: Your answer is not python/Matlab code, but literal 8x8 matrices that have zeros at the same locations. Using ellipses "…" is not allowed. (10 points)

(5) Let G be a geometric random variable with continuation rate q. That is, $pmf_G(n) = q^n(1-q)$. What is H(G)? (10 points)

(6) Let δ be a small number and $q = \exp(-\delta)$. What is the limit of $\mathrm{cdf}_G(t/\delta)$ as $\delta \to 0$? Which distribution's cdf coincide with that limit? (10 points)

(7) What is the limit of $H(G) + \ln(\delta)$ as $\delta \to 0$? (10 points)

(8) What is the differential entropy of the random variable you got in question (6)? Does your answer coincide with (7)? (10 points)

In fact, the differential entropy of a continuous random variable is always the limit of $H(G) + \ln(\delta)$ as $\delta \to 0$, where G is the discrete approximation.

(9) Newly produced tanks are labeled by consecutive integers while old tanks are retired. We therefore assume that, at any given time, the tanks the enemy have are labeled by the set of integers a, $a + 1$, … b, where we do not know a and b. And we are interested in learning b – a, the number of tanks available to the enemy.

Suppose that [a, b] = [100, 200], what is the probability that five random tanks all lie in the range [120, 150]? Accordingly, if we capture tanks no. 120, 125, 130, 140, 150, the p-value of the null hypothesis $[a, b] = [100, 200]$ is this probability. (10 points)

(10) What is the probability that five random tanks from [100, 200] span a range that is shorter than 30? Note that I didn't say the range has to be 120~150. This probability gives the p-value of the null hypothesis $b - a = 100$.

(10 points)

PS: Your final answer should be converted to floating-point numbers.